Math 173 – Assignment #3 **Name:** Solution Set

Total = 60

1. Find the inverse of the following function.

$f(x) = \sqrt[5]{x} - 7$

$\underline{f^{-1}(x) = (x+7)^5}$

replace: $y = \sqrt[5]{x} - 7$

swap: $x = \sqrt[5]{y} - 7$

solve: $x + 7 = \sqrt[5]{y}$

$(x+7)^5 = y$

replace: $f^{-1}(x) = (x+7)^5$

2

2. Find the inverse of the following function, and state the inverse's domain and range.

$f(x) = \dfrac{x+1}{x-3}$ ← domain of f: denom ≠ 0

$\underline{f^{-1}(x) = \dfrac{3x+1}{x-1}}$ 2

replace: $y = \dfrac{x+1}{x-3}$

swap: $x = \dfrac{y+1}{y-3}$

solve: $x(y-3) = y+1$

$xy - 3x = y+1$

$xy - y = 3x+1$

$y(x-1) = 3x+1$

$y = \dfrac{3x+1}{x-1}$

replace: $f^{-1}(x) = \dfrac{3x+1}{x-1}$

domain of f^{-1}: denom ≠ 0

domain: $\underline{\{x \mid x \neq 1\}}$ 1

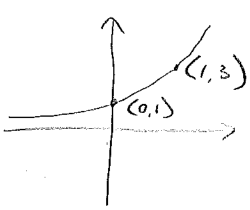
range: $\underline{\{y \mid y \neq 3\}}$ 1

recall that range of f^{-1} is domain of f

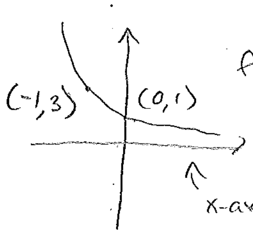
3. Sketch the graphs of the following functions. Include at least two accurate points in your sketch and also indicate the location of any asymptotes.

4

a) $f(x) = 3^{-x} + 1$



$f(x) = 3^x$

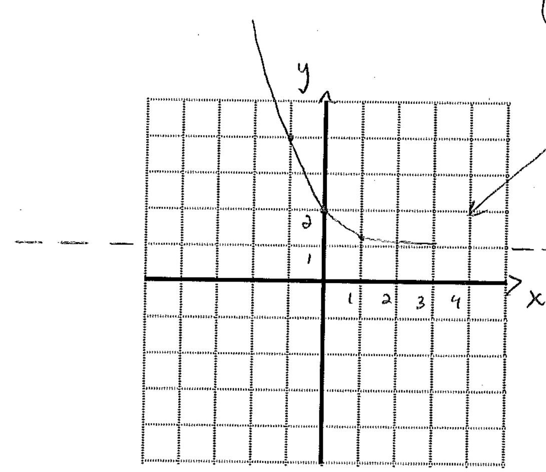


$f(x) = 3^{-x}$

(flip across y)

x-axis is asymptote

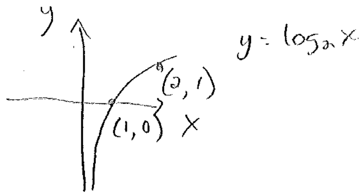
so shift up by 1

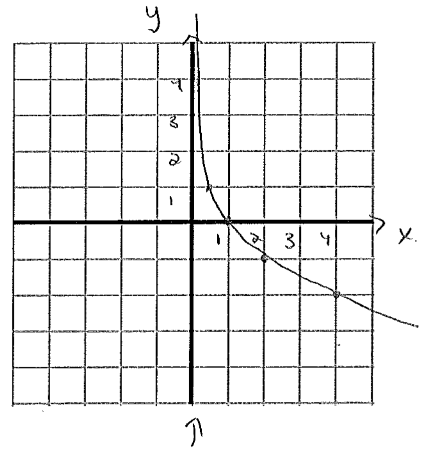


gets close to y = 1 but does not touch

y = 1 is asymptote

b) $f(x) = -\log_2(x)$





x = 0 is asymptote

3

4. Use composition of functions to show that the following functions are inverses.

$$f(x) = \frac{1}{2x} + 1, \; f^{-1}(x) = \frac{1}{2x-2}$$

4

$(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= f\left(\frac{1}{2x-2}\right)$

$= \frac{1}{2\left(\frac{1}{2x-2}\right)} + 1$

$= \frac{2x-2}{2} + 1$

$= x - 1 + 1$

$= x$ ✓

$(f^{-1} \circ f)(x) = f^{-1}(f(x))$

$= f^{-1}\left(\frac{1}{2x} + 1\right)$

$= \frac{1}{2\left(\frac{1}{2x} + 1\right) - 2}$

$= \frac{1}{\frac{1}{x} + 2 - 2}$

$= \frac{1}{1/x}$

$= x$ ✓

5. Find each of the following. Give exact answers.

a) $\log_3 \frac{1}{81}$ $= \log_3 3^{-4}$ -4 1

b) $\ln\sqrt{e}$ $= \ln e^{1/2}$ $1/2$ 1

c) $\log_{81} 3$ $= \log_{81} (81)^{1/4}$ $1/4$ 1

d) $\log_x 1$ $= \log_x x^0$ 0 1

e) $\log_x(-1)$ undefined   1

cannot be ≤ 0

6. Calculate using the base-change formula.

a) $\log_{82} 0.381$ $= \frac{\log 0.381}{\log 82} = -0.218974$ -0.219 1

b) $\log_\pi 142$ $= \frac{\log 142}{\log \pi} = 4.32925$ 4.329 1

(or can use ln's here)

7. Express in terms of $\ln a$, $\ln b$, and $\ln c$.

a) $\ln\sqrt{\frac{a^5}{bc^3}}$ $= \ln \frac{a^{5/2}}{b^{1/2}c^{3/2}}$ $\frac{5}{2}\ln a - \frac{1}{2}\ln b - \frac{3}{2}\ln c$ 2

$= \ln a^{5/2} - \ln b^{1/2} - \ln c^{3/2}$

$= \frac{5}{2}\ln a - \frac{1}{2}\ln b - \frac{3}{2}\ln c$

b) $\ln\frac{\sqrt{b}}{e^2}$ $= \ln\sqrt{b} - \ln e^2$ $\frac{1}{2}\ln b - 2$ 2

$= \frac{1}{2}\ln b - 2$

8. Simplify.

a) $\log_a \sqrt{ax} - \log_a \dfrac{a}{\sqrt{x}}$ $\log_a x - \frac{1}{2}$ (3)

$\log_a \dfrac{\sqrt{ax}}{a/\sqrt{x}}$

$\log_a \sqrt{ax} \cdot \dfrac{\sqrt{x}}{a}$

$\log_a \dfrac{x}{\sqrt{a}}$

$\log_a x - \log_a \sqrt{a}$

$\log_a x - \frac{1}{2}$

b) $\log_3 \sqrt[m]{3}$ $\frac{1}{m}$ (1)

c) $\log_y y^{a-157}$ $a-157$ (1)

d) $e^{\ln x + 2\ln y}$ $= e^{\ln x + \ln y^2}$

$= e^{\ln xy^2}$ xy^2 (3)

9. Solve. Give exact answers.

a) $3^{1-4x} = \dfrac{1}{27}$ $\{1\}$ (2)

$3^{1-4x} = 3^{-3}$

$1 - 4x = -3$

$-4x = -4$

$x = 1$

b) $\log_2(x-3)+\log_2(x+3)=4$

$\{5\}$

(3)

$\log_2(x^2-9)=4$

$x^2-9=2^4$

$x^2=16+9=25$

$x=\pm 5$

$=5$

check:

$x=5$

$\log_2 2 + \log_2 8 = 4$

$1 + 3 = 4$ ✓

$x=-5$

$\log_2 \cancel{(-8)}$? extraneous

c) $10^{2x}=7^{x-1}$

$\left\{\frac{-\log 7}{2-\log 7}\right\}$

or $\left\{\frac{\log 7}{\log 7-2}\right\}$

(4)

$\log 10^{2x} = \log 7^{x-1}$

$2x = (x-1)\log 7$

$2x = x\log 7 - \log 7$

$2x - x\log 7 = -\log 7$

$x(2-\log 7) = -\log 7$

$x = \frac{-\log 7}{2-\log 7}$

d) $\log(x^2)-\log(3-x)=\log 4$

$\{-6, 2\}$

(3)

$\log \frac{x^2}{3-x} = \log 4$

$\frac{x^2}{3-x} = 4$

$x^2 = 12-4x$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6, 2$

check:

$x=6$

$\log 36 - \log 9 = \log 4$ ✓

$x=2$

$\log 4 - \log 1 = \log 4$ ✓

e) $\log(\ln x)=0$

$\{e\}$

(2)

$\ln x = 10^0$

$\ln x = 1$

$x = e^1$

$= e$

10. A dish of lasagna baked at 375°F is taken out of the oven at 5 pm into a kitchen that is at 75°F. After 20 minutes, the temperature of the lasagna is 225°F. How long will it take the lasagna to cool down to 100°F? (Use Newton's Law of Cooling, page 402.)

Using initial info to find k:

$T = T_0 + (T_1 - T_0)e^{-kt}$

$225 = 75 + (375 - 75)e^{-k(20)}$

$150 = 300 e^{-k \cdot 20}$

$\frac{1}{2} = e^{-20k}$

$\ln \frac{1}{2} = -20k$

$k = -\frac{\ln \frac{1}{2}}{20}$

≈ 0.034657

now find t for T = 100 °F

$T = T_0 + (T_1 - T_0)e^{-kt}$

$100 = 75 + (375 - 75)e^{-0.034657t}$

$25 = 300 e^{-0.034657t}$

$\frac{1}{12} = e^{-0.034657t}$

$\ln \frac{1}{12} = -0.034657t$

$t = \frac{\ln \frac{1}{12}}{-0.034657}$

$= 71.6993$

≈ 70 min

5

The lasagna will cool to 100°F after 70 minutes

11. As part of a science experiment, Pat is cryogenically frozen for many years. Once she wakes up, she finds that her bank account, which had a balance of ~~$1500~~ $1600 when she was frozen, has grown to 5.8 million dollars. The interest rate on her account has remained at ~~5~~ 6% the entire time. How long has she been asleep if her account was compounded

a) monthly?
b) continuously?

a) monthly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$5.8 \times 10^6 = 1500\left(1 + \frac{0.06}{12}\right)^{12t}$

$\frac{5.8 \times 10^6}{1600} = \left(1 + \frac{0.06}{12}\right)^{12t}$

$\log\left(\frac{5.8 \times 10^6}{1500}\right) = 12t \log\left(1 + \frac{0.06}{12}\right)$

$t = \frac{\log\left(\frac{5.8 \times 10^6}{1500}\right)}{12 \log\left(1 + \frac{0.06}{12}\right)}$

$= 136.935$ years

b) continuously

5

$A = Pe^{rt}$

$5.8 \times 10^6 = 1\,00\, e^{0.06t}$

$\frac{5.8 \times 10^6}{1600} = e^{0.06t}$

$\ln\left(\frac{5.8 \times 10^6}{1600}\right) = 0.0\ t$

$t = \frac{\ln\left(\frac{5.8 \times 10^6}{1600}\right)}{0.06}$

$= 136.593$ years

She's been asleep for 136.9 years for monthly compounding and 136.6 y for continuous compounding.