Math 173 – Assignment #4

Name: Solution Set

Total : 60

1. Convert the angles in radians to degrees and the angles in degrees to radians. Show your work. Leave in terms of π, if appropriate.

   a) $135° = 135°\left(\frac{\pi}{180°}\right) = \frac{3\pi}{4}$ Answer: $3\pi/4$ ($\approx 2.35619 \to (-\frac{1}{2})$)

   b) $-\frac{11\pi}{6} = -\frac{11\pi}{6}\left(\frac{180°}{\pi}\right) = -330°$ Answer: $-330°$ (3)

   c) $-90 = -90\left(\frac{180°}{\pi}\right) = \frac{-16200°}{\pi}$

   $\approx -5156.6°$ Answer: $\frac{-16200°}{\pi}$ or $-5156.6°$

2. Use a calculator to evaluate the following trig functions. Round to two decimal places.

   a) $\csc 10.5 = \frac{1}{\sin 10.5} = -1.13676$ Answer: -1.14

   if calc in DEG, get 5.49

   b) $\sin\left(-\frac{\pi}{9}\right) = -0.34202$ Answer: -0.34 (3)

   if calc in DEG, get -0.01

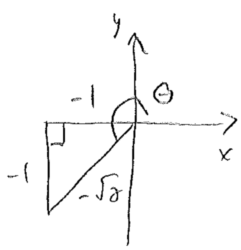
   c) ~~sec 19.3°~~ $\sin^{-1}(0.77) = 50.35°$ or 0.88 rads   Answer: $50.35°$ or 0.88

   output is an angle

3. Find the exact value of following trig function.

$$\sec\left(-\frac{3\pi}{4}\right) = \frac{1}{\cos\left(-3\pi/4\right)}$$

Answer: $-\sqrt{2}$ (2)



$\cos\left(-\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$ so $\sec\left(-\frac{3\pi}{4}\right) = -\sqrt{2}$

4. Simplify. $\dfrac{1}{1-\sin x}+\dfrac{1}{1-\sin(-x)}$

$\dfrac{2}{\cos^2 x}$ or $2\sec^2 x$

$$\frac{1}{1-\sin x}+\frac{1}{1+\sin x}$$

$$\left(\frac{1+\sin x}{1+\sin x}\right)\frac{1}{1-\sin x}+\frac{1}{1+\sin x}\left(\frac{1-\sin x}{1-\sin x}\right)$$

$$\frac{1+\sin x+1-\sin x}{(1+\sin x)(1-\sin x)}$$

$$\frac{2}{1-\sin^2 x}$$

$$\frac{2}{\cos^2 x} \text{ or } 2\sec^2 x$$

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5. Use the sum/difference identities to find an equivalent expression for $\csc\left(x+\dfrac{\pi}{2}\right)$.

$\sec x$ or $\dfrac{1}{\cos x}$

$$\sin(x+\pi/2) = \sin x \cancel{\cos \pi/2}^{\,0} + \cos x \cancel{\sin \pi/2}^{\,1}$$

$$= \cos x$$

3

$$\csc(x+\pi/2) = \frac{1}{\sin(x+\pi/2)} = \frac{1}{\cos x} = \sec x$$

6. Use the sum/difference identities to evaluate sin 105° exactly.

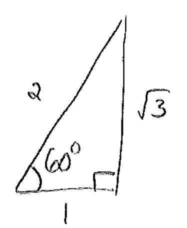
$\dfrac{\sqrt{2}+\sqrt{6}}{4}$ or $\dfrac{\sqrt{2}(1+\sqrt{3})}{4}$

$$\sin 105^\circ = \sin(45^\circ+60^\circ)$$

$$= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2}+\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$



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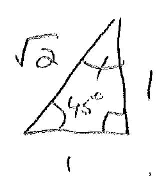


7. Simplify. $\sin \alpha$

$\sin(\alpha - \beta)\cos\beta + \cos(\alpha - \beta)\sin\beta$

long way: $= (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\cos\beta + (\cos\alpha\cos\beta + \sin\alpha\sin\beta)\sin\beta$

$= \sin\alpha\cos^2\beta - \cancel{\cos\alpha\sin\beta\cos\beta} + \cancel{\cos\alpha\cos\beta\sin\beta} + \sin\alpha\sin^2\beta$

$= \sin\alpha\,(\underbrace{\cos^2\beta + \sin^2\beta}_{1})$

$= \sin\alpha$

3

short way $= \sin[(\alpha - \beta) + \beta]$

$= \sin\alpha$

8. Let $x = a\cos\theta$. Substitute into $\sqrt{a^2 - x^2}$ and simplify, so that your answer is a trigonometric function of θ without radicals. Assume that θ is in the first quadrant so all trig functions are positive.

$a\sin\theta$

2

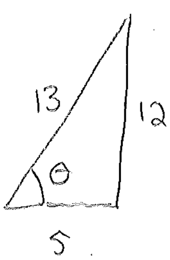
$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\cos^2\theta}$

$= \sqrt{a^2(1 - \cos^2\theta)}$

$= \sqrt{a^2\sin^2\theta}$

$= a\sin\theta$

9. If $\cos\theta = \dfrac{5}{13}$ and θ is in quadrant I, find the exact value of sin 2θ, cos 2θ, and tan 2θ. State also the quadrant in which the angle 2θ lies.



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{5}{13}\right)^2 - 1 = \frac{50}{169} - \frac{169}{169} = \frac{-119}{169}$

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$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{120}{169}\left(-\frac{169}{119}\right) = \frac{-120}{119}$

Since $\sin 2\theta$ is $+$ while $\cos 2\theta$ & $\tan 2\theta$ are $-$, (2θ) must be in QII

10. Prove the following trig identities.

a) difference of cubes! $\longrightarrow$ $$\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=\frac{2+\sin 2\theta}{2}$$

$$\frac{\cancel{(\cos\theta-\sin\theta)}(\cos^2\theta+\cos\theta\sin\theta+\sin^2\theta)}{\cancel{\cos\theta-\sin\theta}} = \frac{2+2\sin\theta\cos\theta}{2}$$

$$(\cos^2\theta+\sin^2\theta)+\cos\theta\sin\theta = 1+\sin\theta\cos\theta$$

$$1+\cos\theta\sin\theta = 1+\sin\theta\cos\theta$$

✓ QED

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b) $$(\sin x+\cos x)(\sec x+\csc x)=\frac{\sec^2 x+2\tan x}{\tan x}$$

$$(\sin x+\cos x)\left(\frac{1}{\cos x}+\frac{1}{\sin x}\right) = \frac{\tan^2 x+1+2\tan x}{\tan x}$$

$$\frac{\sin x}{\cos x}+\frac{\sin x}{\sin x}+\frac{\cos x}{\cos x}+\frac{\cos x}{\sin x} = \frac{\tan^2 x}{\tan x}+\frac{1}{\tan x}+\frac{2\tan x}{\tan x}$$

$$\tan x+1+1+\cot x = \tan x+\cot x+2$$

5

$$\tan x+\cot x+2 = \tan x+\cot x+2$$

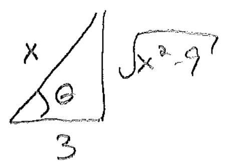
✓

QED

11. Evaluate $\tan\left(\cos^{-1}\frac{3}{x}\right)$.

$\frac{\sqrt{x^2-9}}{3}$

find $\tan\theta$ where $\theta = \cos^{-1}\left(\frac{3}{x}\right)$ (3)



$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{x^2-9}}{3}$

12. Solve, finding all solutions in either [0,360°) or [0,2π).

a) $\sin 2x = \sin x$

$\{0, 60°, 180°, 300°\}$

or $\{0, \pi/3, \pi, \frac{5\pi}{3}\}$

$2\sin x\cos x = \sin x$

$2\sin x\cos x - \sin x = 0$ (5)

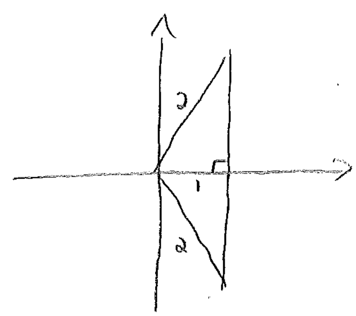
$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ →

$x = 0, 180°$

$x = 60°, 300°$



if say $\sin 2x = 2\sin x$ -3

b) $2\cos^2 x - \cos x = 1$

$\{0, 120°, 240°\}$

$\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

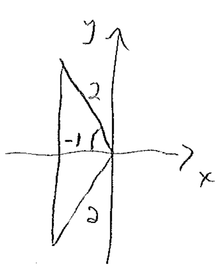
$2\cos^2 x - \cos x - 1 = 0$ $ac = -2$ $(+1 \; -2)$

$2\cos^2 x - 2\cos x + \cos x - 1 = 0$

$2\cos x(\cos x - 1) + (\cos x - 1) = 0$ (5)

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$



$x = 120°, 240°$ $x = 0$

13. Sketch a graph of the function $y = -3\cos(x+\pi)$, and state the function's period, amplitude, domain, and range. Include at least one full period in your sketch.

period: $\frac{2\pi}{|B|} = 2\pi$

amplitude: 3

domain: $\mathbb{R}$

range: $[-3, 3]$

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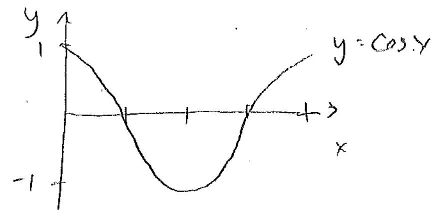

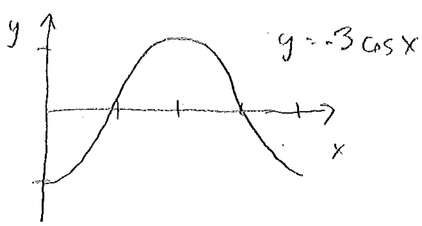

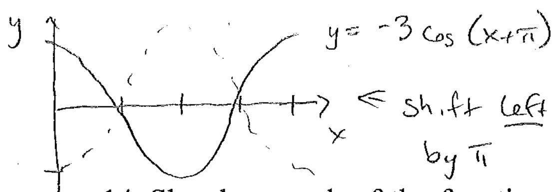

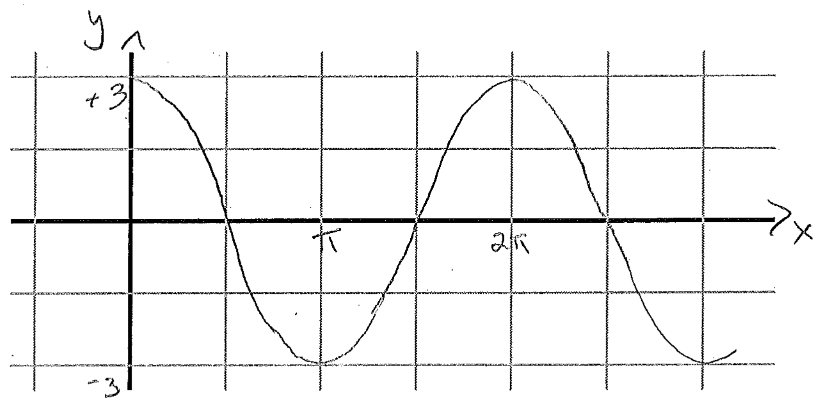

14. Sketch a graph of the function $y = \cot x$, and state the function's period. Include at least two full periods in your sketch. State whether the function is even, odd, or neither.

period: π

odd (symmetric about origin)

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