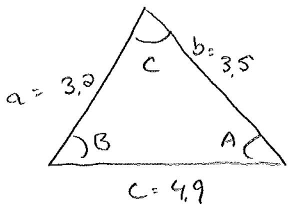
Math 173 – Assignment #5 **Name:** Solution Set

Total: 60

1. Use the information given to solve the following triangles, if possible.

a) $a = 3.2, b = 3.5, c = 4.9$

$A = 41^\circ$, $B = 45^\circ$, $C = 94^\circ$



$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{(3.2)^2 + (3.5)^2 - (4.9)^2}{2(3.2)(3.5)}$

$C = 93.89^\circ \approx 94^\circ$

$\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\sin A = \dfrac{a \sin C}{c}$

$= \dfrac{3.2 \sin 93.89^\circ}{4.9}$

$= 40.66^\circ \approx 41^\circ$

(4)

$B = 180^\circ - A - C = 45.45^\circ$

$\approx 45^\circ$

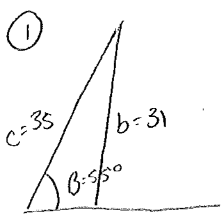
b) $b = 31, c = 35, B = 55^\circ$

either $A = 13^\circ$, $a = 8.3$, $C = 112^\circ$

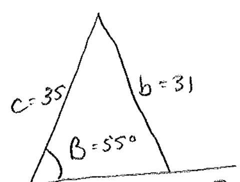
or $A = 57^\circ$, $a = 32$, $C = 68^\circ$

SSA - ambiguous

(1)



or



(2)

$\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

$\sin C = \dfrac{c \sin B}{b} = \dfrac{35 \sin 55^\circ}{31}$

$= 67.65^\circ, 112.35^\circ$

case (2) case (1)

$\approx 68^\circ, 112^\circ$

Case (1):

$A = 180^\circ - B - C$

$= 12.65^\circ \approx 13^\circ$

(6)

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$a = \dfrac{b \sin A}{\sin B} = \dfrac{31 \sin 12.65^\circ}{\sin 55^\circ}$

$= 8.29 \approx 8.3$

Case (2):

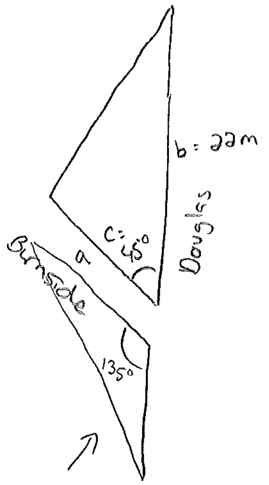
$A = 180^\circ - B - C = 57.35^\circ \approx 57^\circ$

$a = \dfrac{b \sin A}{\sin B} = \dfrac{31 \sin 57.35^\circ}{\sin 55^\circ}$

$= 31.86$

≈ 32

2. Douglas Street runs due north, while Burnside Road branches off from it running northwest. Let's suppose that between these two streets is a nice triangular park with area 133 m^2, and the side running alongside Douglas is 22 m long. How long is the side next to Burnside?



note: if you use this △ instead, get same answer since sin 45° = sin 135°

area = ½ ab sin C

(3)

$a = \frac{2(\text{area})}{b \sin C}$

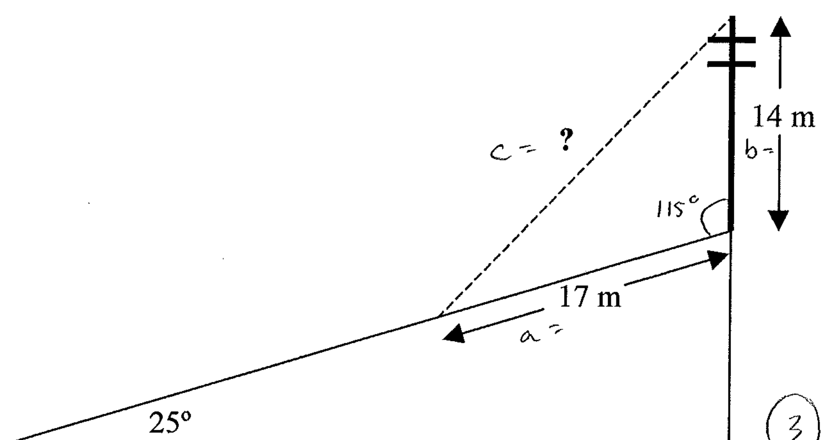
$= \frac{2 \cdot 133}{22 \sin 45°}$

$= 17.0991$ m

$= 17$ m

The side of the park next to Burnside is 17 m long.

3. A hill is inclined 25° to the horizontal. A 14-metre tall telephone pole stands vertically at the top of the hill. How long a wire will it take to reach from the top of the pole to a point 17 metres downhill from the base of the pole?



(3)

$c^2 = a^2 + b^2 - 2ab\cos C$

$= (17)^2 + (14)^2 - 2(17)(14)\cos 115°$

$c = 26.1948$

$= 26$

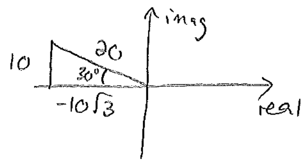
The wire needs to be 26 m long.

4. Change the following complex number into trig form (or $re^{i\theta}$, if you prefer).

a) 18 $\quad$ $18(\cos 0 + i\sin 0)$ or $18e^{i0}$

b) $-5i$ $\quad$ $5(\cos 270° + i\sin 270°)$ or $5e^{i3\pi/2}$

c) $-10\sqrt{3}+10i$ $\quad$ $20(\cos 150° + i\sin 150°)$ or $20e^{i5\pi/6}$



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5. Change the following from trig form back to standard form ($a+bi$). Give exact answers.

a) $5(\cos 60° + i\sin 60°) = 5\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ $\quad$ $\frac{5}{2} + \frac{5i\sqrt{3}}{2}$

4

b) $3(\cos\pi + i\sin\pi) = 3(-1 - 0i) = -3$ $\quad$ -3

6. Divide the two numbers using whatever method you wish.

$$\frac{18\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}{3\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)} = \frac{18e^{i\pi/2}}{3e^{i3\pi/4}}$$ $\quad$ $6e^{i(-\pi/4)}$

4

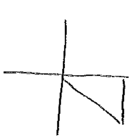
$= 6e^{i(\pi/2 - 3\pi/4)}$

$= 6e^{i(-\pi/4)}$ ← acceptable answer

$= 6(\cos(-\pi/4) + i\sin(-\pi/4))$ ← also okay

$= 6\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$

$= 3\sqrt{2} - 3i\sqrt{2}$ ← also okay



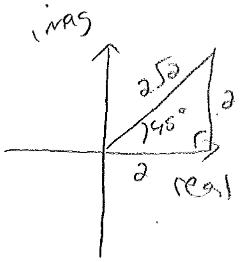
7. Raise the number to the given power. You may leave your answer in whatever form you wish.

$(2+2i)^4 = \left(2\sqrt{2}\, e^{i\pi/4}\right)^4$

$= 2^4 \cdot 2^2\, e^{i\pi}$

$= 16 \cdot 4 \cdot (-1)$

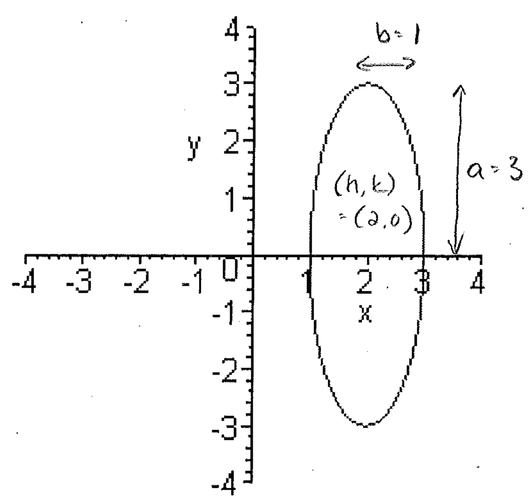
$= -64$



-64 or $64e^{i\pi}$

4

8. Write the equation for the ellipse graphed below. Also, state the coordinates of the foci and all of the vertices. Lastly, calculate the eccentricity.



$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ because "vertically" oriented

Centre at $(h,k) = (2,0)$

$c^2 = a^2 - b^2$

$= 9 - 1$

$c = \sqrt{8} = 2\sqrt{2}$

$ecc = \frac{c}{a} = \frac{2\sqrt{2}}{3}$

equation: $\frac{(x-2)^2}{1} + \frac{y^2}{9} = 1$ 2

foci: $(2, \pm 2\sqrt{2})$ 1

vertices: $(2, \pm 3)$ 1

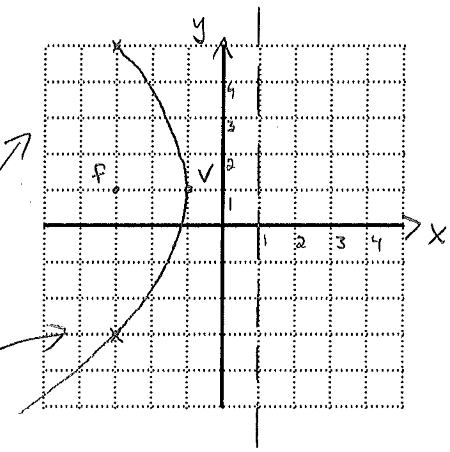
eccentricity: $2\sqrt{2}/3$ 1

9. Write the standard form of the equation for the parabola with vertex at (-1,1) and focus at (-3,1). Sketch the parabola on the graph below (as accurately as possible!), being sure to include the directrix.

$p = -2$ oriented horizontally

$(y-k)^2 = 4p(x-h)$

$(y-1)^2 = -8(x+1)$



these points are 4 units from focus & directrix

equation of parabola: $(y-1)^2 = -8(x+1)$ (2)

(3)

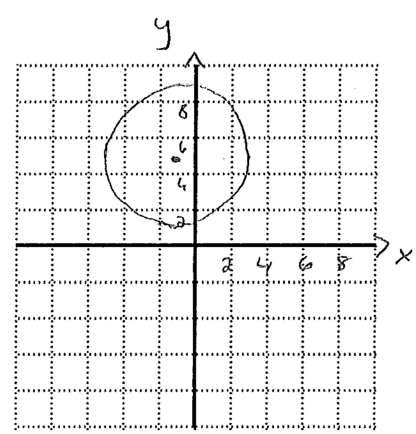
10. Find the centre and radius of the following circle. Sketch the graph as accurately as possible. (5 total)

$x^2 + y^2 + 2x - 10y = -10$

$x^2 + 2x + \underline{1} + y^2 - 10y + \underline{25} = -10 + \underline{1} + \underline{25}$

$(x+1)^2 + (y-5)^2 = 4^2$

(1)



(2)

centre: $(-1, 5)$

radius: 4

(2)

11. Find the centre, vertices, foci, and eccentricity of the following hyperbola. Sketch the graph as accurately as possible, including the asymptotes.

8 total

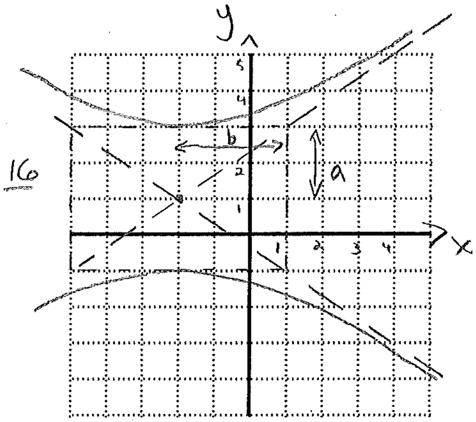
$$9y^2 - 18y - 4x^2 - 16x - 43 = 0$$

$9y^2 - 18y \quad -4x^2 - 16x = 43$

$9(y^2 - 2y + \underline{1}) - 4(x^2 + 4x + \underline{4}) = 43 + \underline{9} - \underline{16}$

$9(y-1)^2 - 4(x+2)^2 = 36$

$\dfrac{(y-1)^2}{2^2} - \dfrac{(x+2)^2}{3^2} = 1$ (1)



(3)

centre: $(-2, 1)$ (1)

foci: $(-2, 1 \pm \sqrt{13})$ (1)

vertices: $(-2, 3)$, $(-2, -1)$ (1)

eccentricity: $\sqrt{13}/2$ (1)

$a = 2$
$b = 3$
$c^2 = a^2 + b^2$
$c = \sqrt{13} \approx 3.6$

12. Complete the square to convert the following equation to the form of a conic section. Which conic section is it (parabola, circle, ellipse, hyperbola)? What are the coordinates of the centre?

5 total

$$9x^2 + 4y^2 - 54x + 16y + 61 = 0$$

$9x^2 - 54x \quad + 4y^2 + 16y = -61$

$9(x^2 - 6x + \underline{9}) + 4(y^2 + 4y + \underline{4})$

$= -61 + \underline{9 \cdot 9} + \underline{4 \cdot 4}$

$9(x-3)^2 + 4(y+2)^2 = -61 + 81 + 16$

$= 36$

$\dfrac{(x-3)^2}{4} + \dfrac{(y+2)^2}{9} = 1$

equation: $\dfrac{(x-3)^2}{2^2} + \dfrac{(y+2)^2}{3^2} = 1$ (2)

which conic? ellipse (1)

centre: $(3, -2)$ (1)