Math 173 – Assignment #6 **Name:** Solution Set

Total: 40

1. Find all terms of the finite sequence $a_n = \frac{(-1)^n}{n!}, 1 \le n \le 4$.

$a_1 = \frac{(-1)^1}{1!} = 1$ $\quad a_3 = \frac{(-1)^3}{3!} = \frac{-1}{6}$

$a_2 = \frac{(-1)^2}{2!} = \frac{1}{2}$ $\quad a_4 = \frac{(-1)^4}{4!} = +\frac{1}{24}$

$-1, \frac{1}{2}, -\frac{1}{6}, \frac{1}{24}$

(2)

2. State whether the following are arithmetic sequences, geometric sequences, or neither. Also, give a formula for the nth term of the sequence.

a) 15, 9, 3, -3, …

$a_1 = 15$

$d = -6$

$a_n = a_1 + (n-1)d$
$= 15 + (n-1)(-6)$
$= 15 - 6n + 6$
$= 21 - 6n$

arithmetic

either $a_n = 21 - 6n$

or $\begin{cases} a_1 = 15 \\ a_n = a_{n-1} - 6 \end{cases}$

(2)

b) $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots \quad \frac{n-1}{n}$

1 2 3 4 5 $\quad$ n

neither

$a_n = \frac{n-1}{n}$

(2)

c) 48, 12, 3, ¾, …

$a_1 = 48$

$r = \frac{1}{4}$

$a_n = a_1 r^{n-1}$
$= 48\left(\frac{1}{4}\right)^{n-1}$

$\left[= 48\left(\frac{1}{4}\right)^n\left(\frac{1}{4}\right)^{-1} \right.$
$\left. = \frac{192}{4^n} \text{ if you insist} \right]$

geometric

either $a_n = 48\left(\frac{1}{4}\right)^{n-1}$

or $\begin{cases} a_1 = 48 \\ a_n = \frac{1}{4} a_{n-1} \end{cases}$

(2)

3. Write the first three terms of the infinite sequence given by the recursion formula

$$\begin{cases} a_1 = 2 \\ a_n = (a_{n-1})^2 + 1 \end{cases}$$

$2, 5, 26, \ldots$ 3

$a_2 = a_1^2 + 1 = 2^2 + 1 = 5$

$a_3 = a_2^2 + 1 = 5^2 + 1 = 26$

4. Find the sum of the following series.

a) $\sum_{j=0}^{4}(3j) = \underset{j=0}{0} + \underset{j=1}{3} + \underset{j=2}{6} + \underset{j=3}{9} + \underset{j=4}{12}$ 30 2

$= 30$

b) $2 + 4 + 6 + \ldots + 88$ 1980 3

arithmetic with

$a = 2$ and $d = 2$

so $S_n = \frac{n}{2}(a_1 + a_n)$

but what's n?

$a_n = a_1 + (n-1)d$

$88 = 2 + (n-1)2$

$86 = 2(n-1)$

$43 = n-1$

$n = 44$

$S_n = \frac{44}{2}(2+88)$

$= 1980$

c) $\sum_{i=0}^{\infty} 300(0.99)^i = \underset{i=0}{300} + \underset{i=1}{300(0.99)} + \underset{i=2}{300(0.99)^2} \ldots$ 30000 2

geometric with

$a_1 = 300$

$r = 0.99$ and $|r| < 1$ ✓

so $S_\infty = \frac{a_1}{1-r} = \frac{300}{1-0.99}$

$= 30000$

d) $\frac{1}{25} - \frac{1}{20} + \frac{1}{16} - \frac{5}{64} + \ldots$ undefined 1

geometric with

$a_1 = 1/25$

$r = -5/4$ ← but $\cancel{|r| < 1}$ not satisfied

5. Find a formula for a_n, given an arithmetic sequence with $a_5 = 30$ and $a_{10} = -5$.

$a_5 = 30$

$a_n = a_1 + (n-1)d$

$30 = a_1 + 4d$

$a_{10} = -5$

$-5 = a_1 + 9d$

system: $30 = a_1 + 4d$ ⇐ mult by -1
$-5 = a_1 + 9d$

$-30 = -a_1 - 4d$
$-5 = a_1 + 9d$
$-35 = 5d$
$d = -7$

so: $30 = a_1 + 4d$
$30 = a_1 - 28$
$a_1 = 58$

$a_n = a_1 + (n-1)d$
$= 58 + (n-1)(-7)$
$= 58 - 7n + 7$
$= 65 - 7n$

$\boxed{a_n = 65 - 7n \text{ or } \begin{cases} a_1 = 58 \\ a_n = a_{n-1} - 7 \end{cases}}$

(5)

6. Expand the following expressions.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

a) $(1-i)^5 = (1)^5 - 5(1)^4 i + 10(1)^3 i^2 - 10(1)^2 i^3 + 5(1) i^4 - i^5$

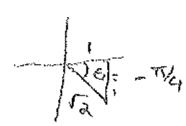
$= 1 - 5i + 10i^2 - 10i^3 + 5i^4 - i^5$

$= 1 - 5i - 10 + 10i + 5 - i$

$\boxed{= -4 + 4i}$

(3)

alternatively: $(1-i)^5 = (\sqrt{2}\, e^{-i\pi/4})^5$
$= \sqrt{2}^5\, e^{-i5\pi/4}$
$= 4\sqrt{2}\, e^{-i5\pi/4}$



b) $\left(x-\sqrt{2}\right)^6$

$= x^6 - 6x^5\sqrt{2} + 15x^4\sqrt{2}^2 - 20x^3\sqrt{2}^3 + 15x^2\sqrt{2}^4 - 6x\sqrt{2}^5 + \sqrt{2}^6$

(3)

$\boxed{= x^6 - 6\sqrt{2}x^5 + 30x^4 - 40\sqrt{2}x^3 + 60x^2 - 24\sqrt{2}x + 8}$

7. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x)=x^3$. $3x^2+3xh+h^2$

$f(x+h)=(x+h)^3 = x^3+3x^2h+3xh^2+h^3$ 3

$$\frac{f(x+h)-f(x)}{h} = \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= \frac{3x^2h+3xh^2+h^3}{h}$$

$$= 3x^2+3xh+h^2$$

8. Find the 4^{th} term in the expansion of $(3x+1)^{12}$. $4330260x^9$

$k=3$ $n=12$

coefficient is $_nC_k = \frac{n!}{k!(n-k)!} = \frac{12!}{3!\,9!} = 220$ 3

term is $_nC_k\,(3x)^{n-k}(1)^k$

$= 220\,(3x)^9(1)^3$

$= 220\cdot 3^9\cdot x^9$

$= 4330260x^9$

9. On November 1, an English teacher had his class read five pages of Tolstoy's <u>War and Peace</u>. He then told them that the number of pages they should read each day should be three more than the day before. If they follow the teacher's instructions, how many pages in total will they have read by November 30th? If the paperback has 1904 pages, will they have finished the book?

arithmetic series with

$a_1 = 5$

$d = 3$

$n = 30$

4

$$S_n = \frac{n}{2}(a_1+a_n) = \frac{n}{2}(2a_1+(n-1)d)$$

$$= \frac{30}{2}(2\cdot 5+29\cdot 3)$$

$$= 1455$$

Students will have read 1455 pages, but will not have finished the book.