CAMOSUN COLLEGE

Math 173 Practice Final

Department of Mathematics

Name (please print clearly): Solution Set Mark:

Signature: ______________ $\overline{}$ 100

Instructor: Patricia Wrean

Instructions:

- Please fill out the above information but do not open this examination until told to begin. When told to begin, check that your exam is complete. There should be 14 pages in total, including this title page. This final exam is 3 hours long.
- Only ordinary scientific (i.e. non-graphing and non-programmable) calculators are allowed.
- Show all of your work. Full marks will only be given when work is shown.

GOOD LUCK!

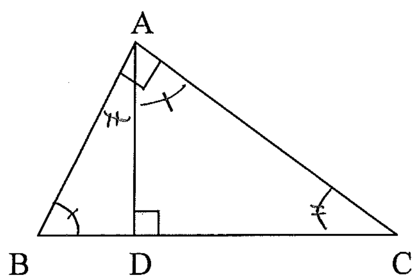
1. Use your calculator to find the values of the following expressions. Round your answers to two decimal places. (3 points)

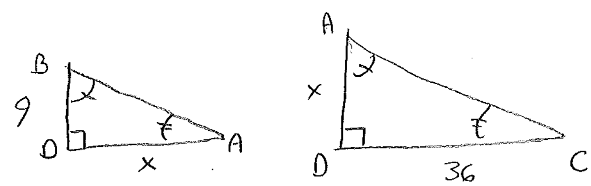
   a) $\log_3 74 = \dfrac{\log 74}{\log 3} = 3.91773$ 3.92 ①

   b) $\sin\dfrac{\pi}{12} = 0.258819$ 0.26 ①

   c) $\cos^{-1}(0.2) = 1.36944$ rads $= 78.463°$ 1.37 or 78.46° ①

2. BD has length 9 and CD has length 36. Find the length of AD. Show your work. (3 points)





$\triangle BDA \sim \triangle ADC$ by AAA ①

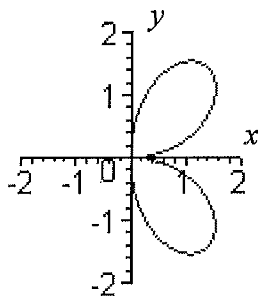
$\dfrac{BD}{AD} = \dfrac{DA}{DC}$ ①

$\dfrac{9}{x} = \dfrac{x}{36}$ so AD = 18 ①

$x^2 = 324$

$x = 18$ (must be +)

3. Consider the graph below. State whether the graph is symmetric with respect to the x-axis, the y-axis, and/or the origin. Also state whether y is a function of x for this graph. (2 points)



symmetric wrt x-axis (but not y or origin) ①

not a function (fails vertical line test) ①

4. Find the vertex of the following parabola and the equation of the axis of symmetry. Sketch the graph of this parabola. (4 points)

$f(x) = -3x^2 + 6x + 1$

either complete square or find vertex

① completing square:

$f(x) = -3(x^2 - 2x \quad) + 1$

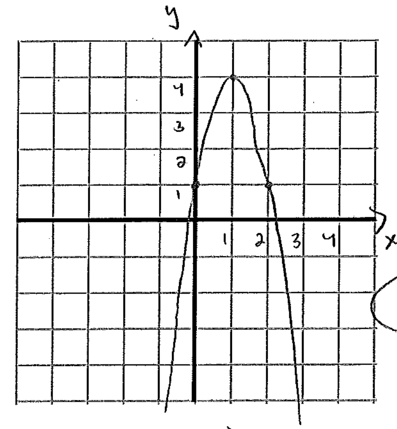
$= -3(x^2 - 2x + 1) + 1 + 3$

$= -3(x-1)^2 + 4$

② or find vertex

$x = -\frac{b}{2a} = \frac{-6}{2(-3)} = 1$

$f(1) = -3(1)^2 + 6 + 1 = 4$



note: y-int is (0,1)

②

vertex: $(1, 4)$ ①

axis of symmetry: $x = 1$ ①

5. Find the inverse of the function $f(x) = \frac{2x}{x+1}$. Also, state the domain and range of $f(x)$. (3 points)

$f(x) = \frac{2x}{x+1}$ ← domain $\{x \mid x \neq -1\}$

$y = \frac{2x}{x+1}$ replace

$x = \frac{2y}{y+1}$ swap

$x(y+1) = 2y$ solve

$xy + x = 2y$

$x = 2y - xy$

$x = y(2-x)$

→ $y = \frac{x}{2-x}$

$f^{-1}(x) = \frac{x}{2-x}$ ← so $x \neq 2$, range of $f(x)$

$f^{-1}(x) = \frac{x}{2-x}$

domain: $\{x \mid x \neq -1\}$

range: $\{x \mid x \neq 2\}$

6. Is the function $f(x) = \frac{1}{x} - x^3$ even, odd, or neither even nor odd? (2 points)

$$\left.\begin{aligned} f(-x) &= \frac{1}{-x} - (-x)^3 \\ &= -\frac{1}{x} + x^3 \end{aligned}\right\} ①$$

$= -\left(\frac{1}{x} - x^3\right)$

$= -f(x)$ ∴ odd ①

odd

7. Consider the polynomial $f(x) = 4x^3 - 16x^2 + 13x - 3$. (6 points)

a) Use the Rational Root Theorem to list all possible rational roots of $f(x)$.

$\frac{\pm p}{\pm q} = \frac{\pm 1, 3}{\pm 1, 2, 4} = \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

1

b) Use Descartes' Rule to list the number of possible positive and negative rational roots of $f(x)$.

$f(x)$ has 3 changes of sign

positive: 3, 1 1

negative: 0 1

$f(-x) = -4x^3 - 16x^2 - 13x - 3$

no changes of sign

c) Factor $f(x)$ completely.

$f(x) = (x-3)(2x-1)^2$

3

$f(1) = 4 - 16 + 13 - 3 \neq 0$

$f(3) = 4 \cdot 27 - 16 \cdot 9 + 13 \cdot 3 - 3 = 0$

$$\begin{array}{r} 4x^2 - 4x + 1 \\ x-3 \overline{) 4x^3 - 16x^2 + 13x - 3} \\ 4x^3 - 12x^2 \\ \hline -4x^2 + 13x \\ -4x + 12x \\ \hline x - 3 \end{array}$$

$4x^2 - 4x + 1 = (2x-1)^2$

so $f(x) = (x-3)(4x^2 - 4x + 1)$
$= (x-3)(2x-1)^2$

8. Solve the following equations. Give exact answers. (8 points)

a) $2m^2 + 3m + 2 = 0$

$\left\{ \frac{-3 \pm i\sqrt{7}}{4} \right\}$

2

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{9 - 16}}{2 \cdot 2}$

$= \frac{-3 \pm \sqrt{-7}}{4}$

$= \frac{-3 \pm i\sqrt{7}}{4}$

b) $4^{3x-1} = e^x$ $\left\{ \dfrac{\ln 4}{3\ln 4 - 1} \right\}$

$\ln 4^{3x-1} = \ln e^x$

$(3x-1)\ln 4 = x$

$3x \ln 4 - \ln 4 = x$

$3x \ln 4 - x = \ln 4$

$x(3\ln 4 - 1) = \ln 4$

$x = \dfrac{\ln 4}{3\ln 4 - 1}$

3

c) $\log_3(x+4) + \log_3(x-4) = 2$ $\{5\}$

$\log_3 (x+4)(x-4) = 2$

$\log_3 (x^2 - 16) = 2$

$x^2 - 16 = 3^2$

$x^2 - 16 = 9$

$x^2 = 25$

$x = \cancel{\pm 5}$

$= +5$

check:

$x = 5$

$\log_3(9) + \log_3(1) = 2$

$2 + 0 = 2$ ✓

$x = -5$

$\log_3(-1)$

domain error so $x = -5$ is extraneous

3

9. Write the following expression as a single logarithm. (2 points)

$2\log_a b - 3\log_a c + \frac{1}{2}\log_a 3$ $\log_a\left(\dfrac{\sqrt{3}\, b^2}{c^3}\right)$ or $\log_a\left(\dfrac{3^{1/2} b^2}{c^3}\right)$

$\log_a b^2 - \log_a c^3 + \log_a \sqrt{3}$

$\log_a\left(\dfrac{b^2\sqrt{3}}{c^3}\right)$

2

10. Given $f(x)=x^3$ and $g(x)=\dfrac{2}{x-1}$, find the following. (4 points)

a) $(g\circ f)(x) = g(f(x))$

$$= \frac{2}{f(x)-1}$$

$$= \frac{2}{x^3-1}$$

$(g \circ f)(x) = \dfrac{2}{x^3-1}$ 2

b) $(f/g)(x) = \dfrac{x^3}{2/x-1}$

$$= x^3 \cdot \frac{x-1}{2}$$

$$= \frac{x^3(x-1)}{2}$$

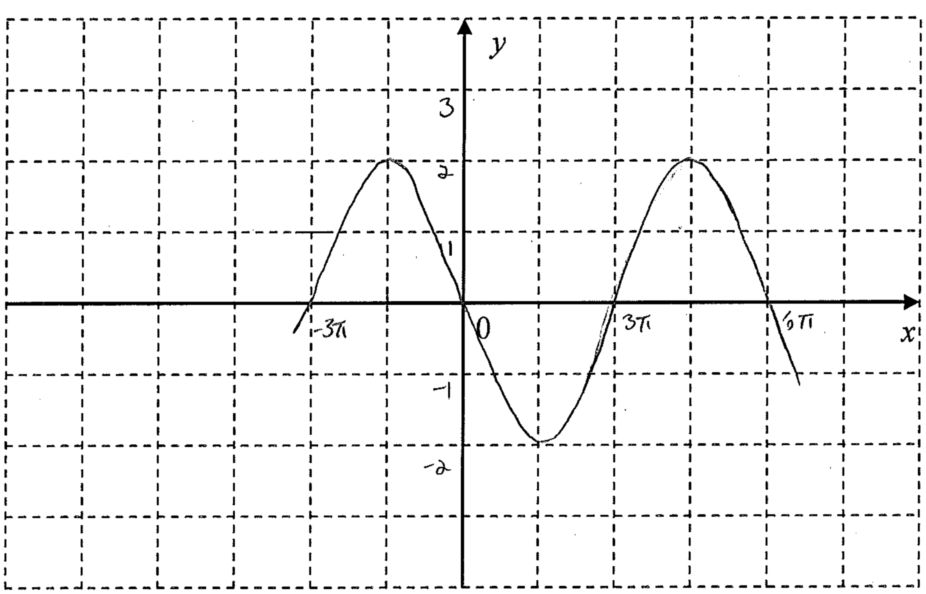
$(f/g)(x) = \dfrac{x^3(x-1)}{2}$ 2

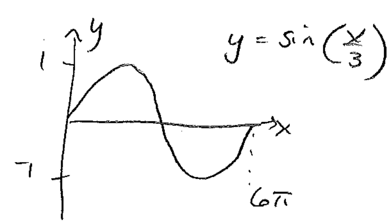
or $\dfrac{x^4-x^3}{2}$

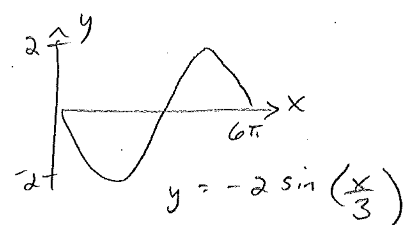
11. Find the amplitude and the period of the following function and sketch the graph over at least one period. (4 points)

$y=-2\sin\left(\dfrac{x}{3}\right)$

amplitude: 2 1, period: $\dfrac{2\pi}{|B|} = \dfrac{2\pi}{1/3} = 6\pi$ 1

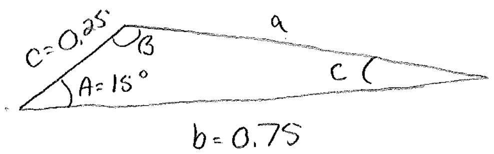






12. Solve the triangle ABC with $b = 0.75$, $c = 0.25$, and $A = 15°$. (4 points)

$a = 0.51$, $B = 158°$, $C = 7°$



$a^2 = b^2 + c^2 - 2bc\cos A$

$= (0.75)^2 + (0.25)^2 - 2(0.75)(0.25)\cos 15°$

$a = 0.512619$

choosing to find C because then don't need to worry which value to pick for B:

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\sin C = \frac{c \sin A}{a}$

$= \frac{0.25 \sin 15°}{0.512619}$

$C = 7.25°$

then $B = 180° - C - A = 158°$

13. Solve the equation in [0,360°) or [0,2π). (4 points)

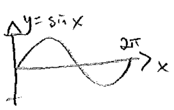
$$\cos 2x \sin x + \sin x = 0$$

$\{0, 90°, 180°, 270°\}$

or $\{0, \pi/2, \pi, 3\pi/2\}$

$\sin x(\cos 2x + 1) = 0$

$\sin x = 0$ or $\cos 2x = -1$



$x = 0, 180°$

$= 0, \pi$

two ways to solve:

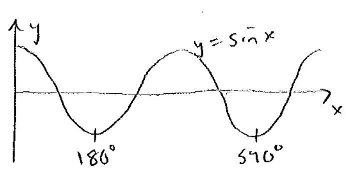
method ①

$\cos 2x = -1$

let $\theta = 2x$

θ in $[0, 720°)$

must remember this part!



$\theta = 180°, 540°$

$x = \frac{\theta}{2} = 90°, 270°$

$= \frac{\pi}{2}, \frac{3\pi}{2}$

method 2:

$\cos 2x = -1$

$2\cos^2 x - 1 = -1$

$2\cos^2 x = 0$

$\cos^2 x = 0$

$\cos x = 0$

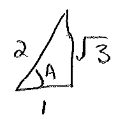
$x = 90°, 270°$

$= \pi/2, \frac{3\pi}{2}$

14. Evaluate the following expression exactly. (4 points)

$$\cos\left(\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{3}{5}\right).$$

$\cos(A+B)$ where $A=\cos^{-1}\left(\frac{1}{2}\right)$ and $B=\cos^{-1}\frac{3}{5}$



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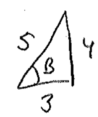


1

$$\cos(A+B)=\cos A\cos B-\sin A\sin B \quad 1$$
$$=\frac{1}{2}\cdot\frac{3}{5}-\frac{\sqrt{3}}{2}\cdot\frac{4}{5}$$
$$=\frac{3-4\sqrt{3}}{10} \quad 1$$

15. Simplify (4 points)

$$\frac{1+\sin 2x+\cos 2x}{1+\sin 2x-\cos 2x}$$

note: choose identities for $\cos 2x$ that will cancel the 1's

$$\frac{1+\sin 2x+2\cos^2 x-1}{1+\sin 2x-(1-2\sin^2 x)}$$

$$\frac{\sin 2x+2\cos^2 x}{\sin 2x+2\sin^2 x}$$

$$\frac{2\sin x\cos x+2\cos^2 x}{2\sin x\cos x+2\sin^2 x}$$

$$\frac{\cancel{2}\cos x\,\cancel{(\sin x+\cos x)}}{\cancel{2}\sin x\,\cancel{(\cos x+\sin x)}}$$

$$\frac{\cos x}{\sin x}$$

$$\boxed{\cot x}$$

16. Prove the identity. (3 points)

$\tan x + \cot x = \csc x \sec x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x} \frac{1}{\cos x}$$

$$\left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right) = \frac{1}{\sin x \cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x} \quad \text{QED}$$

17. Expand $(1 - 3i)^4$. (3 points)

can use any method:

$$(1-3i)^4 = 1^4 - 4\cdot 1^3(3i) + 6\cdot 1^2(3i)^2 - 4\cdot 1\cdot(3i)^3 + (3i)^4$$

$$= 1 - 12i + 54i^2 - 108i^3 + 81i^4$$

$$= 1 - 12i - 54 + 108i + 81$$

$$= 28 + 96i$$

(using $re^{i\theta}$ would also work, though you might get some rounding errors)

18. Is the sequence 48, –12, 3, –¾, … arithmetic, geometric or neither? Find a formula for the n[th] term a_n. (3 points)

geometric

$a_n = 48\left(-\frac{1}{4}\right)^{n-1}$

or recursively
$\begin{cases} a_1 = 48 \\ a_n = -\frac{1}{4} a_{n-1} \end{cases}$

$\left.\begin{array}{l} \frac{a_2}{a_1} = \frac{-12}{48} = -\frac{1}{4} \\ \frac{a_3}{a_2} = \frac{3}{-12} = -\frac{1}{4} \end{array}\right\}$ geometric with $r = -\frac{1}{4}$

$a_n = a_1 r^{n-1}$
$= 48\left(-\frac{1}{4}\right)^{n-1}$

[nitpicker alert:
if you like, you could simplify: $a_n = 48\left(-\frac{1}{4}\right)^n\left(-\frac{1}{4}\right)^{-1}$
$= 48\left(-\frac{1}{4}\right)^n(-4)$
$= -192\left(-\frac{1}{4}\right)^n$
but it's not necessary]

19. Find the sum of the following series. (4 points)

$36+41+46+51+\ldots+381$

$S_{70} = 14595$

series is arithmetic with $a_1 = 36$ and $d = 5$

how many terms?

$a_n = a_1 + (n-1)d$
$381 = 36 + (n-1)5$
$345 = (n-1)5$
$69 = n-1$
$n = 70$

$S_n = \frac{n}{2}(a_1 + a_n)$
$= \frac{70}{2}(36 + 381)$
$= 14595$

20. Write the following series in sigma notation. (2 points)

$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots$

$\sum_{k=1}^{\infty} \frac{1}{k+1}$

or $\sum_{k=2}^{\infty} \frac{1}{k}$

or equivalent

$a_n = \frac{1}{n+1}$

so series is $\sum_{k=1}^{\infty} \frac{1}{k+1}$

or $\sum_{k=2}^{\infty} \frac{1}{k}$

21. Consider the graph of the rational function $f(x) = \dfrac{x^2 - 4}{x^2 - 1}$. (9 points)

a) Find the coordinates of all x-intercepts (if any). (2, 0) and (-2, 0) 1

set num = 0

$x^2 - 4 = 0$

$x = \pm 2$

b) Find the coordinates of all y-intercepts (if any). (0, 4) 1

set x = 0 $f(0) = \frac{-4}{-1} = 4$

c) Find the equations of all vertical asymptotes (if any). $x = \pm 1$ 1

set denom = 0

$x^2 - 1 = 0$

$x = \pm 1$

d) Find the equations of all horizontal asymptotes (if any). $y = 1$ 1

degree num = degree of denom so ratio of leading coeffs

$y = \frac{1}{1} = 1$

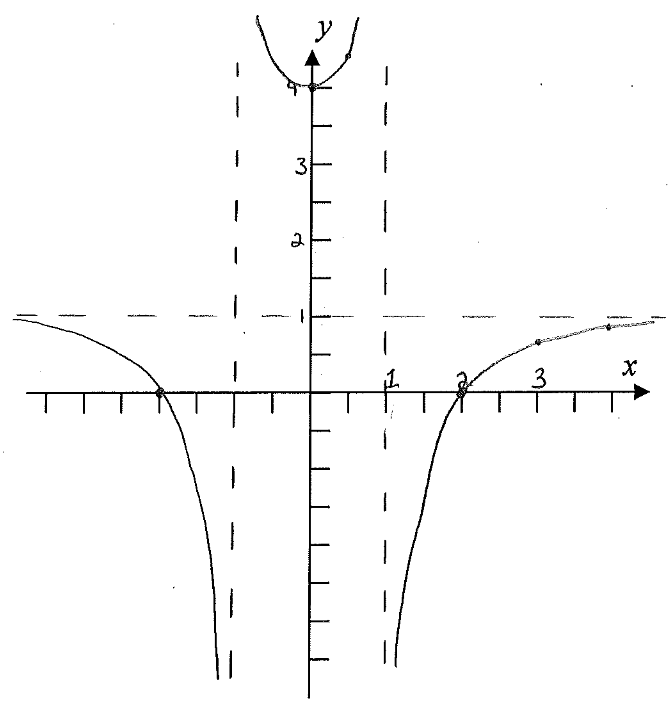
e) Find the equations of all oblique asymptotes (if any). none 1

none

f) Sketch the graph of $y = f(x)$ as accurately as possible. Identify all intercepts and asymptotes on your graph.

extra points:

x	y
½	5
3	5/8
4	4/5



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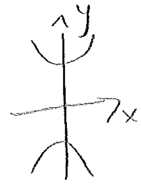
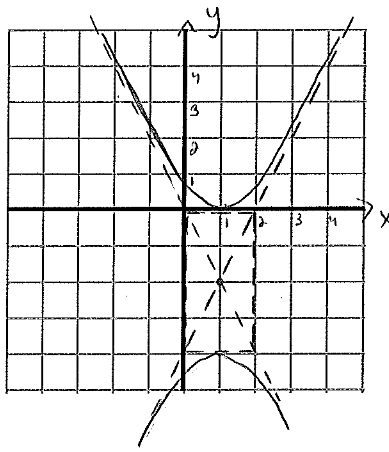
22. Find the centre and the foci of the following hyperbola. Sketch the graph as accurately as possible. (4 points)

$$\frac{(y+2)^2}{4}-\frac{(x-1)^2}{1}=1$$



$a=2$ $h=1$

$b=1$ $k=-2$



2

$a^2+b^2=c^2$

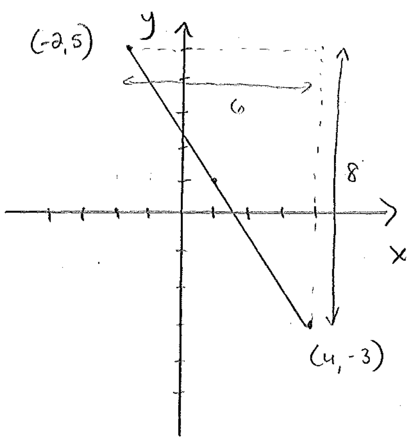
$4+1=c^2$

$c=\sqrt{5}$

centre: $(1,-2)$ 1

foci: $(1,-2\pm\sqrt{5})$ 1

23. The diameter of a circle has endpoints at (–2, 5) and (4, –3). What is the equation of the circle? (3 points)



the centre of the circle will be at the midpoint, (1, 1) (can find this either by counting squares or by the midpoint formula)

1

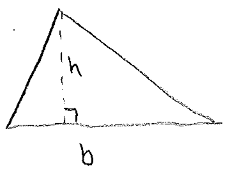
the length of the diameter is 10 units by Pythagorus,

so radius is 5 1

$(x-h)^2+(y-k)^2=r^2$

$\boxed{(x-1)^2+(y-1)^2=25}$ 1

24. The sum of the base and the height of a triangle is 18 cm. Find the maximum area for this triangle, and also state the dimensions of the triangle which give this maximum area. (4 points)



$b + h = 18$

$$\text{Area} = \tfrac{1}{2} bh$$
$$= \tfrac{1}{2} b(18-b)$$
$$= 9b - \tfrac{1}{2}b^2$$
$$= -\tfrac{1}{2}b^2 + 9b$$

this is a parabola with vertex at

$$\frac{-b}{2a} = \frac{-9}{2\cdot(-\frac{1}{2})} = +9$$

so $b=9$, $h=9$, and $\text{Area} = \tfrac{1}{2}\,9\cdot 9 = \frac{81}{2}$

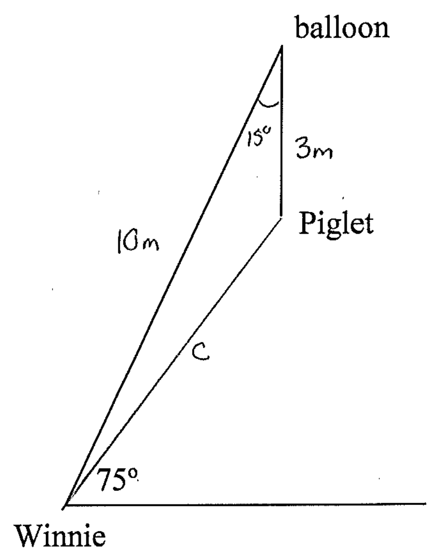
The base and height are both 9 cm and the maximum area is $81/2$ cm^2.

25. Winnie the Pooh is flying a helium balloon whose string is 10 metres long. Because of a breeze, the line makes an angle of 75° with respect to the ground. Dangling 3 metres directly below the balloon is Piglet. How far away from Winnie is Piglet? (Give the straight-line distance, not the horizontal distance.) (5 points)

Cosine law:

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 3^2 + 10^2 - 2(3)(10)\cos 15^\circ$$
$$c = 7.14454$$

Piglet is 7.1 m away from Winnie.



26. In Smallville, Lex Luthor is threatening to destroy Superman's superpowers using Red Kryptonite. However, Lex's plans have misfired – Red Kryponite is radioactive and after 11.5 days, 88% has decayed away, leaving only 12% of the original material. What is the half-life of Red Kryptonite? (4 points)

$$A = A_0 e^{-rt}$$

find r:

$$0.12 A_0 = A_0 e^{-r(11.5)}$$

$$0.12 = e^{-11.5r}$$

$$\ln 0.12 = -11.5r$$

$$r = \frac{\ln 0.12}{-11.5}$$

$$\approx 0.184371$$

find t (halflife):

$$\tfrac{1}{2} A_0 = A_0 e^{-0.184371t}$$

$$\tfrac{1}{2} = e^{-0.184371t}$$

$$\ln \tfrac{1}{2} = -0.184371t$$

$$t = \frac{\ln \frac{1}{2}}{-0.184371}$$

$$\approx 3.75953$$

The halflife is 3.76 days.