



Math 173 Practice Final

Department of Mathematics

Name (please print clearly): _____

Mark:

Signature: _____

100

Instructor: Patricia Wrean

Instructions:

- Please fill out the above information but do not open this examination until told to begin. When told to begin, check that your exam is complete. There should be 14 pages in total, including this title page. This final exam is 3 hours long.
- Only ordinary scientific (i.e. non-graphing and non-programmable) calculators are allowed.
- Show all of your work. Full marks will only be given when work is shown.

GOOD LUCK!

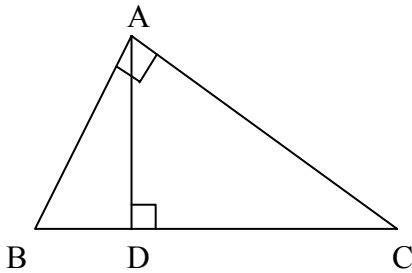
1. Use your calculator to find the values of the following expressions. Round your answers to two decimal places. (3 points)

a) $\log_3 74$ _____

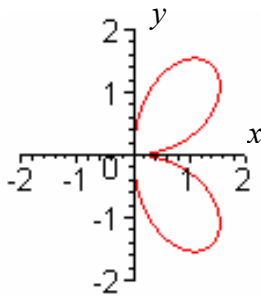
b) $\sin \frac{\pi}{12}$ _____

c) $\cos^{-1}(0.2)$ _____

2. BD has length 9 and CD has length 36. Find the length of AD. Show your work. (3 points)

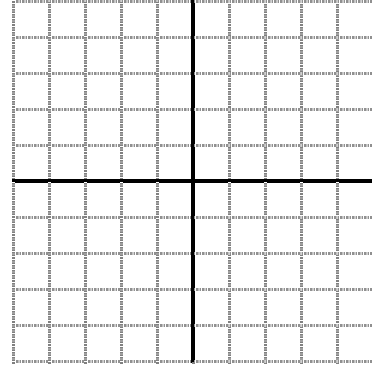


3. Consider the graph below. State whether the graph is symmetric with respect to the x -axis, the y -axis, and/or the origin. Also state whether y is a function of x for this graph. (2 points)



4. Find the vertex of the following parabola and the equation of the axis of symmetry. Sketch the graph of this parabola. (4 points)

$$f(x) = -3x^2 + 6x + 1$$



vertex: _____

axis of symmetry: _____

5. Find the inverse of the function $f(x) = \frac{2x}{x+1}$. Also, state the domain and range of $f(x)$. (3 points)

domain: _____

range: _____

6. Is the function $f(x) = \frac{1}{x} - x^3$ even, odd, or neither even nor odd? (2 points)

7. Consider the polynomial $f(x) = 4x^3 - 16x^2 + 13x - 3$. (6 points)

a) Use the Rational Root Theorem to list all possible rational roots of $f(x)$.

b) Use Descartes' Rule to list the number of possible positive and negative rational roots of $f(x)$.

positive: _____
negative: _____

c) Factor $f(x)$ completely.

8. Solve the following equations. Give exact answers. (8 points)

a) $2m^2 + 3m + 2 = 0$

b) $4^{3x-1} = e^x$

c) $\log_3(x+4) + \log_3(x-4) = 2$

9. Write the following expression as a single logarithm.

(2 points)

$$2\log_a b - 3\log_a c + \frac{1}{2}\log_a 3$$

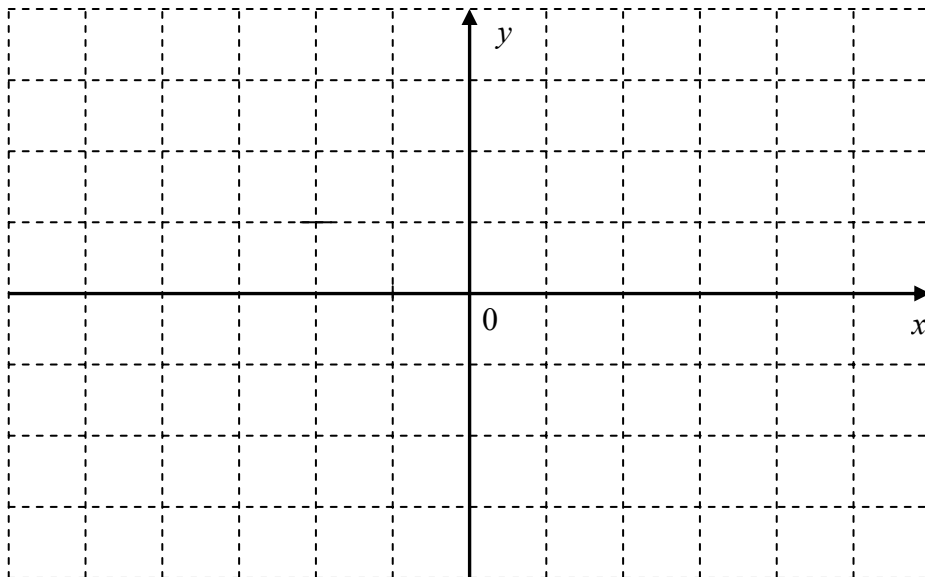
10. Given $f(x) = x^3$ and $g(x) = \frac{2}{x-1}$, find the following. (4 points)

a) $(g \circ f)(x)$ _____

b) $(f/g)(x)$ _____

11. Find the amplitude and the period of the following function and sketch the graph over at least one period. (4 points)

$y = -2\sin\left(\frac{x}{3}\right)$ amplitude: _____, period: _____



12. Solve the triangle ABC with $b = 0.75$, $c = 0.25$, and $A = 15^\circ$. (4 points)

13. Solve the equation in $[0, 360^\circ)$ or $[0, 2\pi)$. (4 points)

$$\cos 2x \sin x + \sin x = 0$$

14. Evaluate the following expression exactly.

(4 points)

$$\cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{3}{5}\right)$$

15. Simplify

(4 points)

$$\frac{1 + \sin 2x + \cos 2x}{1 + \sin 2x - \cos 2x}$$

16. Prove the identity.

(3 points)

$$\tan x + \cot x = \csc x \sec x$$

17. Expand $(1 - 3i)^4$.

(3 points)

18. Is the sequence $48, -12, 3, -\frac{3}{4}, \dots$ arithmetic, geometric or neither? Find a formula for the n^{th} term a_n . (3 points)

19. Find the sum of the following series. (4 points)

$$36 + 41 + 46 + 51 + \dots + 381$$

20. Write the following series in sigma notation. (2 points)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$$

21. Consider the graph of the rational function $f(x) = \frac{x^2 - 4}{x^2 - 1}$. (9 points)

a) Find the coordinates of all x-intercepts (if any). _____

b) Find the coordinates of all y-intercepts (if any). _____

c) Find the equations of all vertical asymptotes (if any). _____

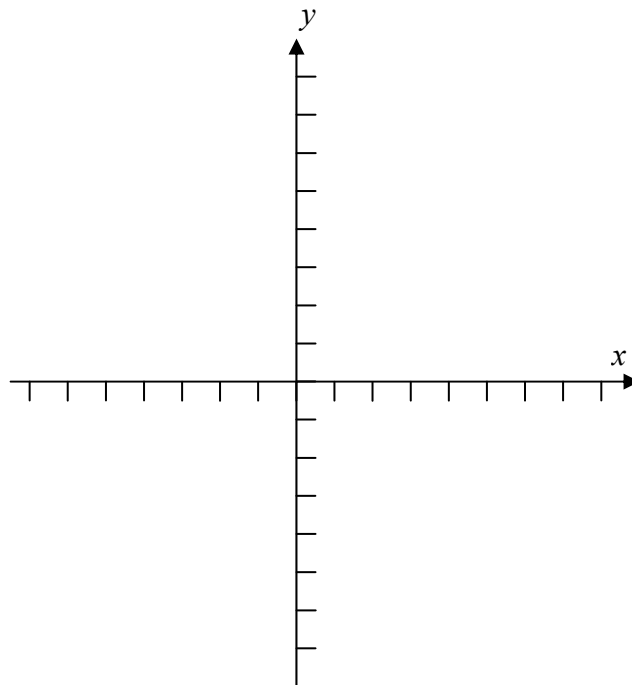
d) Find the equations of all horizontal asymptotes (if any). _____

e) Find the equations of all oblique asymptotes (if any). _____

f) Sketch the graph of $y = f(x)$ as accurately as possible. Identify all intercepts and asymptotes on your graph.

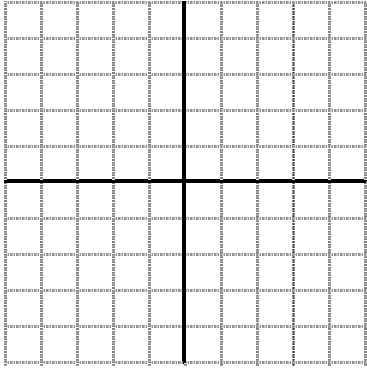
extra points:

x	y
$\frac{1}{2}$	5
3	$\frac{5}{8}$
4	$\frac{4}{5}$



22. Find the centre and the foci of the following hyperbola. Sketch the graph as accurately as possible. (4 points)

$$\frac{(y+2)^2}{4} - \frac{(x-1)^2}{1} = 1$$



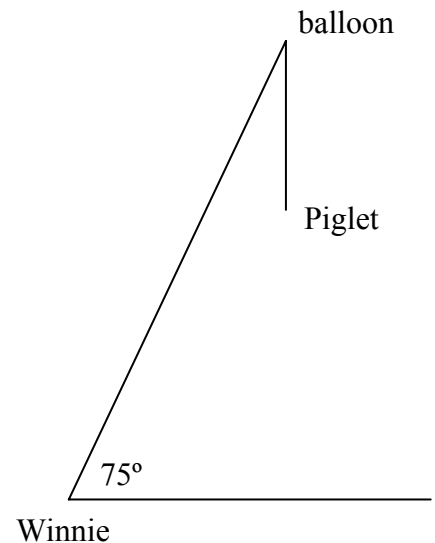
centre: _____

foci: _____

23. The diameter of a circle has endpoints at $(-2, 5)$ and $(4, -3)$. What is the equation of the circle? (3 points)

24. The sum of the base and the height of a triangle is 18 cm. Find the maximum area for this triangle, and also state the dimensions of the triangle which give this maximum area. (4 points)

25. Winnie the Pooh is flying a helium balloon whose string is 10 metres long. Because of a breeze, the line makes an angle of 75° with respect to the ground. Dangling 3 metres directly below the balloon is Piglet. How far away from Winnie is Piglet? (Give the straight-line distance, not the horizontal distance.) (5 points)



26. In Smallville, Lex Luthor is threatening to destroy Superman's superpowers using Red Kryptonite. However, Lex's plans have misfired – Red Kryptonite is radioactive and after 11.5 days, 88% has decayed away, leaving only 12% of the original material. What is the half-life of Red Kryptonite? (4 points)