Math 173 – Rules for Integral and Rational Exponents

Let a and b be real numbers and m and n be integers. Then the following rules for exponents apply.

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

$$\frac{1}{a^{-n}} = a^n$$

$$a^{-1} = \frac{1}{a}$$

$$a^0 = 1$$

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(a^m\right)^n = a^{mn}$$

$$\left(ab\right)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(a^m\right)^{1/n}$$

Exercises

Simplify.

1. $3^{15} \cdot 3^{-3}$
2. $-5a \cdot 6a^{-12}$
3. $-\frac{1}{2} w^{-4} \cdot \left(-6w^{-2}\right)$
4. $\frac{x^{-2}}{x^3}$
5. $\frac{2r^{-3}t^{-1}}{10r^5t^2t^{-3}}$
6. $\left(1+2^{-1}\right)^{-2}$
7. $2^{-2}\left(\frac{1}{4}\right)^{-3}$
8. $\frac{\left(-2x^{-5}y\right)\left(-3xy^6\right)}{-6x^{-6}y^2}$
9. $\left(x^{-2}\right)^3\left(x^{-3}\right)^{-2}$
10. $\left(2x^{-1}y^2\right)^{-3}$
11. $\frac{\left(2a^{-2}b\right)^{-3}}{\left(2ab^{-1}\right)^2}\left(2a^2b^{-7}\right)$
12. $\left(-8\right)^{-5/3}$

13. $\left(\frac{2^6}{3^9}\right)^{-1/3}$

14. $16^{3/2}$

15. $x^{1/2}x^{1/4}$

16. $125^{2/3}$

17. $\left(\frac{9}{16}\right)^{-1/2}$

18. $\left(5x^{-1/2}\right)^{-2}$

Solutions

1. 3^{12}

2. $-\frac{30}{a^{11}}$ or $-30a^{-11}$

3. $\frac{3}{w^6}$ or $3w^{-6}$

4. x^{-5} or $\frac{1}{x^5}$

5. $\frac{1}{5r^8}$ or $\frac{r^{-8}}{5}$

6. $\frac{4}{9}$

7. 16

8. $-x^2y^5$

9. 1

10. $\frac{x^3}{8y^6}$

11. $\frac{a^6}{16b^8}$

12. $-\frac{1}{32}$

13. $\frac{27}{4}$

14. 64

15. $x^{3/4}$

16. 25

17. $\frac{4}{3}$

18. $\frac{x}{25}$