

Math 173 – Section 4.5: Sketching Rational Functions

1. Consider the following rational function: $f(x) = \frac{4}{x-2}$.

What is the y-intercept? set $x=0$ $(0, -2)$

$$y = f(0) = \frac{4}{-2} = -2$$

What are the x-intercepts? none

set numerator to zero
and solve for x

~~$4=0$~~ \therefore no x-intercepts

Are there any vertical asymptotes? If so, where?

set denominator to zero $\rightarrow x-2=0$

so $x=2$

yes,
at $x=2$

Are there any horizontal asymptotes? If so, where?

degree of num $<$ degree of denom

yes at $y=0$

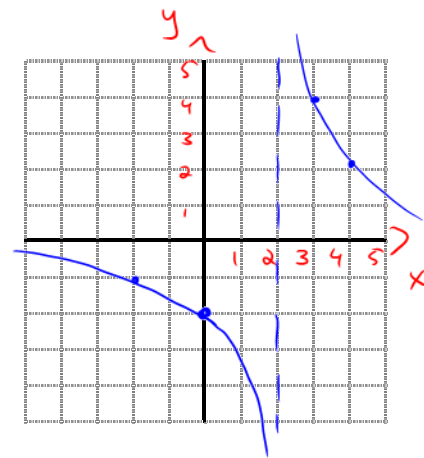
Are there any oblique asymptotes? If so, where?

no

Sketch the graph as accurately as possible.

extra points

x	y
-2	-1
3	4
4	2



$$f(x) = \frac{4}{x-2}$$

2. Consider the following rational function: $f(x) = \frac{3x-1}{x}$.

What is the y-intercept? set $x=0$ none

$$y = f(0) = \frac{-1}{0} = \text{undefined}$$

What are the x-intercepts? set num = 0 $(\frac{1}{3}, 0)$

$$3x-1=0$$

$$x = \frac{1}{3}$$

Are there any vertical asymptotes? If so, where? set denom = 0

$$x = 0$$

yes, at $x=0$

Are there any horizontal asymptotes? If so, where?

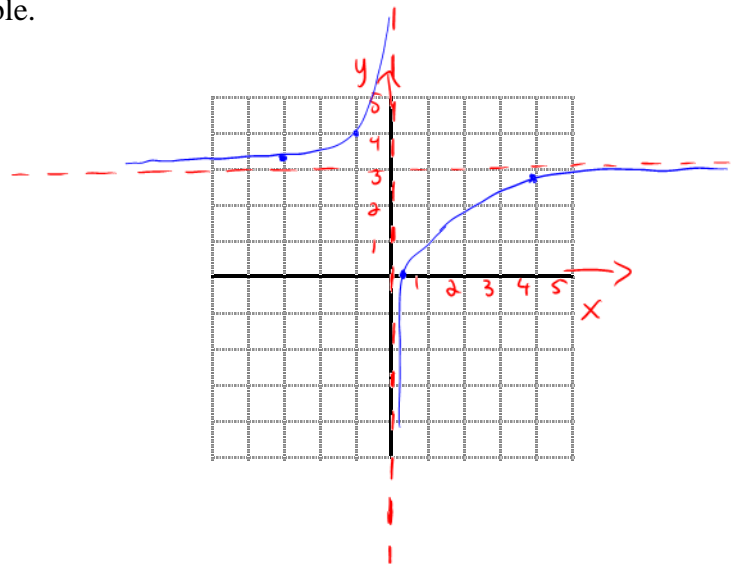
yes, at $y=3$

Are there any oblique asymptotes? If so, where?

no

Sketch the graph as accurately as possible.

x	y
-3	$3\frac{1}{3}$
-1	4
4	$\frac{1}{4} = 2\frac{3}{4}$



3. Consider the following rational function: $f(x) = \frac{x}{x^2 - x - 2} = \frac{x}{(x-2)(x+1)}$

What is the y-intercept? set $x = 0$ (0, 0)

$$y = f(0) = \frac{0}{-2} = 0$$

What are the x-intercepts? (0, 0)

$$\text{set num} = 0$$

$$x = 0$$

Are there any vertical asymptotes? If so, where? set denom = 0

$$(x-2)(x+1) = 0$$

yes, at $x = 2$ and $x = -1$

Are there any horizontal asymptotes? If so, where?

degree of num < degree denom

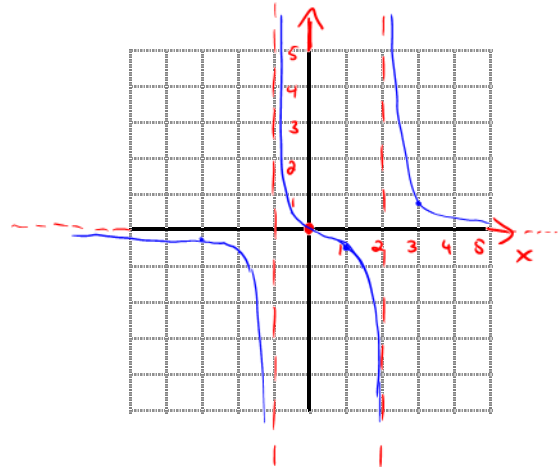
yes, at $y = 0$

Are there any oblique asymptotes? If so, where?

no

Sketch the graph as accurately as possible.

x	y
-3	-0.3
-2	-0.5
$-\frac{1}{2}$	0.4
1	-0.5
3	0.75



4. Consider the following rational function: $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$.

What is the y-intercept?

$$y = f(0) = \frac{2(-9)}{-4} = \frac{9}{2}$$

(0, 4.5)

What are the x-intercepts?

$$x = \pm 3$$

(3, 0), (-3, 0)

Are there any vertical asymptotes? If so, where?

$$x = \pm 2$$

yes, at $x = \pm 2$

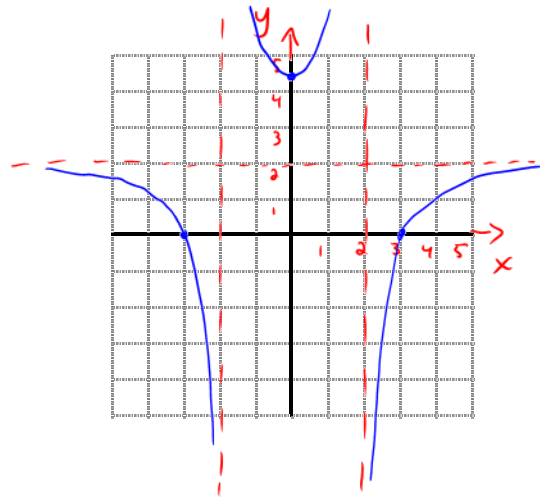
Are there any horizontal asymptotes? If so, where?

$$\text{yes, at } y = 2$$

Are there any oblique asymptotes? If so, where?

no

Sketch the graph as accurately as possible.



5. Consider the following rational function: $f(x) = \frac{x^2 - x}{x + 1}$.

What is the y-intercept? $\text{set } x=0$ (0,0)
 $y=0$

What are the x-intercepts? $\text{set num} = 0$ (0,0), (1,0)
 $x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$

Are there any vertical asymptotes? If so, where? $\text{set denom} = 0$
 $x + 1 = 0 \Rightarrow \boxed{x = -1}$

Are there any horizontal asymptotes? If so, where?

no

Are there any oblique asymptotes? If so, where?

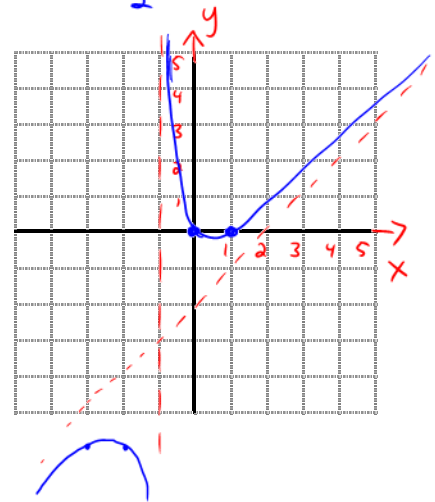
yes

(degree num > degree of denom)

Sketch the graph as accurately as possible.

$$\begin{array}{r} x-2 \\ x+1 \overline{) x^2 - x + 0} \\ \underline{x^2 + x} \\ -2x + 0 \\ \underline{-2x - 2} \\ 2 \end{array} \quad \boxed{y = x - 2}$$

x	y
-2	-6
-3	-6



6. Consider the following rational function: $f(x) = \frac{x^2 - x - 2}{x - 1}$.

What is the y-intercept? set $x=0$

$$y = \frac{-2}{-1} = 2$$

(0, 2)

What are the x-intercepts? set num = 0

$$\begin{aligned} x^2 - x - 2 &= 0 \\ (x + 1)(x - 2) &= 0 \quad x = 2, -1 \end{aligned}$$

(2, 0), (-1, 0)

Are there any vertical asymptotes? If so, where? set denom = 0

$$x - 1 = 0$$

$$\boxed{x = 1}$$

Are there any horizontal asymptotes? If so, where?

no

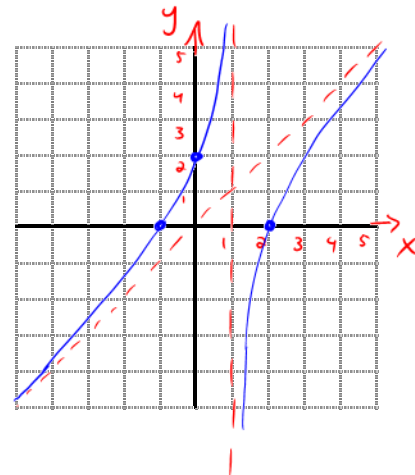
Are there any oblique asymptotes? If so, where?

yes

$$\begin{array}{r} x \\ x-1 \overline{) x^2 - x - 2} \\ \underline{x^2 - x} \\ -2 \end{array}$$

yes, at $\boxed{y = x}$

Sketch the graph as accurately as possible.



7. Consider the following rational function: $f(x) = \frac{x}{x^2 + 1}$.

What is the y-intercept? set $x = 0$

$$y = 0$$

(0, 0)

What are the x-intercepts? set num = 0

$$x = 0$$

(0, 0)

Are there any vertical asymptotes? If so, where? set denom = 0

$$x^2 + 1 = 0$$

no

Are there any horizontal asymptotes? If so, where?

yes, at $y = 0$

Are there any oblique asymptotes? If so, where?

no

Sketch the graph as accurately as possible.

x	y
1	$\frac{1}{2} = 0.5$
2	$\frac{2}{5} = 0.4$
3	$\frac{3}{10} = 0.3$
4	$\frac{4}{17} \approx 0.235$
-1	-0.5
-2	-0.4
-3	-0.3

