Math 173 – Section 5.6: Newton's Law of Cooling

If you take a freshly-brewed cup of coffee and put it into a room at a normal temperature, the coffee will immediately start cooling towards that room temperature. Similarly, if you place an ice cube into a room, the ice cube will warm up until it reaches the melting point of the ice (the thermodynamics of the ice melting are beyond the scope of this course). And, once the ice is melted, the resulting water will continue to warm towards room temperature.

These are examples of Newton's Law of Cooling, which gives the relationship between the temperature of an object and the temperature of the object's surroundings as a function of time. If the object is initially at temperature T_1 and is placed into surroundings at temperature T_0, the object's temperature at some later time t will be given by

$$T(t) = T_0 + \left(T_1 - T_0\right)e^{-kt}$$

where k is the rate at which the object warms/cools.

Example

Let's suppose that a cup of coffee at temperature 60ºC is placed into a room at 20ºC, and after 15 minutes, the temperature of the coffee is found to be 25ºC. Calculate k for this cup of coffee.

Answer

To find the rate of cooling, you would take the following quantities:

$$T_0 = 20^\circ\text{C}, T_1 = 60^\circ\text{C}, T = 25^\circ\text{C}, t = 15 \text{ minutes}$$

and substitute them into the equation above

$$\begin{aligned}
&T(t) = T_0 + \left(T_1 - T_0\right)e^{-kt} \\
&25 = 20 + \left(60 - 20\right)e^{-k(15)} \\
&5 = 40e^{-15k} \\
&\frac{5}{40} = e^{-15k} \\
&\ln\left(\frac{5}{40}\right) = -15k \\
&k = -\frac{1}{15}\ln\left(\frac{5}{40}\right) \approx 0.138629
\end{aligned}$$

and the units of k would be "per minute" to cancel the "minutes" unit in t.
