

Math173 – Supplement to Section 8.3

1. Change the following complex numbers to the form $re^{i\theta}$.

- a) -7
- b) $6i$
- c) $4 - 3i$
- d) $\sqrt{2} + i\sqrt{2}$
- e) $12\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

2. Change the following complex numbers to the form $a+bi$.

- a) $5e^{i\pi}$
- b) $12e^{-i\frac{\pi}{3}}$
- c) $2e^{\frac{i\pi}{2}}$

3. Multiply or divide the following complex numbers.

- a) $(5e^{i\pi})(3e^{i\pi})$
- b) $\left(12e^{-i\frac{\pi}{3}}\right)\left(4e^{\frac{i\pi}{2}}\right)$
- c) $(\sqrt{2} + i\sqrt{2})(1 - i\sqrt{3})$
- d) $\left(12e^{-i\frac{\pi}{3}}\right) \div \left(4e^{\frac{i\pi}{2}}\right)$
- e) $\frac{(\sqrt{2} + i\sqrt{2})}{(1 - i\sqrt{3})}$

4. Raise the following complex numbers to the given power and simplify. Write your answer in the form $a+bi$.

- a) $(5e^{i\pi})^4$
- b) $\left(12e^{-i\frac{\pi}{3}}\right)^9$
- c) $\left(2e^{\frac{i\pi}{2}}\right)^7$
- d) $(\sqrt{2} + i\sqrt{2})^8$

Solutions

1. a) $r=7, \theta=180^\circ$ or π , so answer is $7e^{i\pi}$ (or any other angle coterminal with π)

b) $r=6, \theta=90^\circ$, so answer is $6e^{i\frac{\pi}{2}}$ (or any other coterminal angle)

c) $r=5, \theta = -37^\circ$ (or -0.64 rads), so answer is $5e^{-0.64i}$

d) $r=2, \theta=45^\circ$ or $\pi/4$, so answer is $2e^{i\frac{\pi}{4}}$

e) $r=12, \theta=\frac{3\pi}{2}$, so answer is $12e^{i\frac{3\pi}{2}}$

2. a) $5e^{i\pi} = 5 \cos \pi + 5i \sin \pi = 5(-1) + 5i(0) = -5$

b) $6 - 6i\sqrt{3}$

c) $2i$

3. a) $(5e^{i\pi})(3e^{i\pi}) = 15e^{i2\pi} = 15$

b) $\left(12e^{-i\frac{\pi}{3}}\right)\left(4e^{i\frac{\pi}{2}}\right) = 48e^{i\left(-\frac{\pi}{3}+\frac{\pi}{2}\right)} = 48e^{i\frac{\pi}{6}}$

c) $(\sqrt{2} + i\sqrt{2})(1 - i\sqrt{3}) = (2e^{i\pi/4})(2e^{-i\pi/3}) = 4e^{i\left(\frac{\pi}{4}-\frac{\pi}{3}\right)} = 4e^{-i\left(\frac{\pi}{12}\right)}$

d) $\left(12e^{-i\frac{\pi}{3}}\right) \div \left(4e^{i\frac{\pi}{2}}\right) = 3e^{i\left(-\frac{\pi}{3}-\frac{\pi}{2}\right)} = 3e^{-i\frac{5\pi}{6}}$

e) $\frac{(\sqrt{2} + i\sqrt{2})}{(1 - i\sqrt{3})} = \frac{(2e^{i\pi/4})}{(2e^{-i\pi/3})} = 1e^{i\left(\frac{\pi}{4}+\frac{\pi}{3}\right)} = 1e^{i\left(\frac{7\pi}{12}\right)}$

4. a) $(5e^{i\pi})^4 = 625e^{i4\pi} = 625$

b) $\left(12e^{-i\frac{\pi}{3}}\right)^9 = 12^9 e^{-i3\pi} = -12^9$

c) $\left(2e^{i\frac{\pi}{2}}\right)^7 = 128e^{i\frac{7\pi}{2}} = -128i$

d) $(\sqrt{2} + i\sqrt{2})^8 = \left(2e^{i\frac{\pi}{4}}\right)^8 = 256e^{i2\pi} = 256$