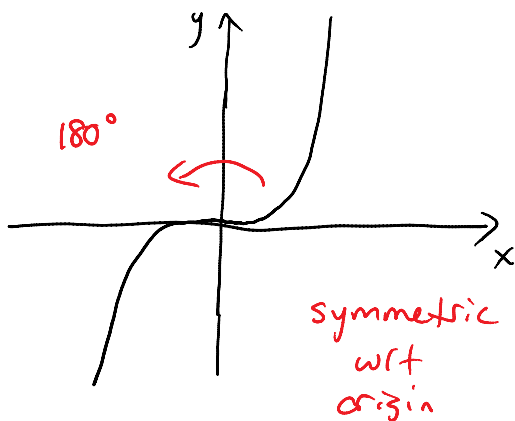
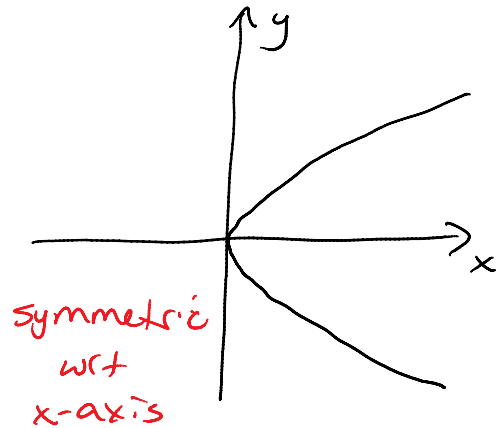
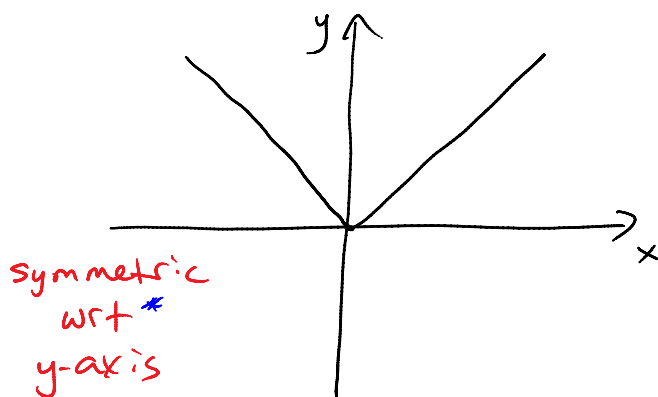


Section 2.4: Symmetry and Transformations

Monday, January 18, 2016
10:32 AM

symmetry - a regular, balanced arrangement on opposite sides of a line or around a centre / axis

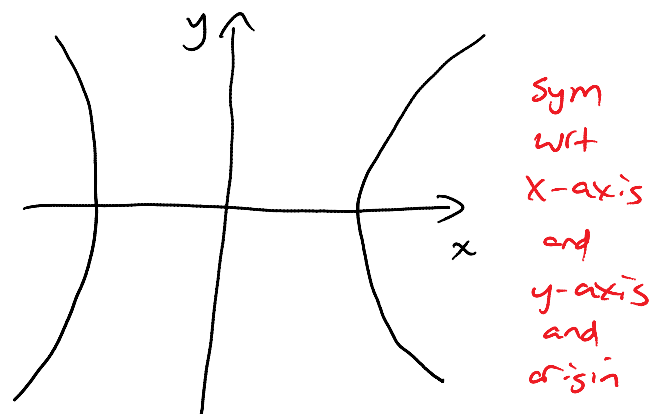
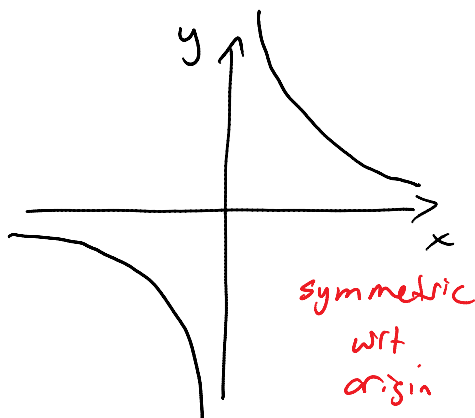
types of symmetry:



wrt = "with respect to"

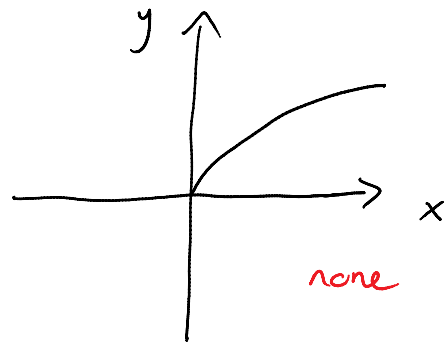
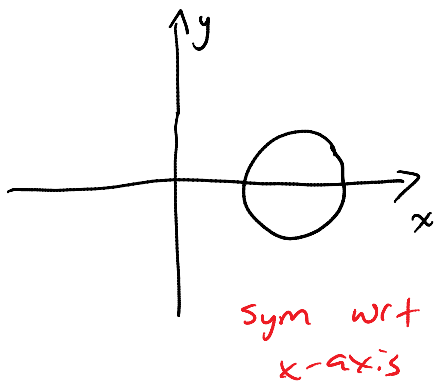
← or can flip in x, then y

example: state the type of symmetry, if any.



|| wrt origin

/ | \ and origin



algebraic tests for symmetry:

- if replacing y by $-y$ gives an equivalent equation, symmetric wrt x -axis
- if replacing x by $-x$ gives an equivalent equation, symmetric wrt y -axis
- if replacing x by $-x$ and y by $-y$ gives an equivalent equation, and symmetric wrt origin

example: Use the algebraic tests for symmetry on the relation $x = y^2$ to determine the symmetry

x -axis: replace y by $-y$

$$\begin{aligned} x &= y^2 \\ x &= (-y)^2 && \leftarrow \text{replace} \\ &= y^2 && \leftarrow \text{simplify} \end{aligned}$$

same!

\therefore sym wrt x -axis

y-axis: replace x by $-x$

$$\begin{aligned} x &= y^2 \\ -x &= y^2 \end{aligned} \quad \leftarrow \text{replace}$$

not same!

\therefore not sym wrt y-axis

origin: replace both

$$\begin{aligned} x &= y^2 \\ -x &= (-y)^2 \\ -x &= y^2 \end{aligned} \quad \leftarrow \text{replace}$$

not same

\therefore not sym wrt origin

note: test $y = x^3$ wrt origin

$$\begin{aligned} y &= x^3 \\ -y &= (-x)^3 \\ -y &= -x^3 \end{aligned} \quad \begin{aligned} &\leftarrow \text{replace} \\ &\leftarrow \text{simplify} \end{aligned}$$

\therefore equivalent

\therefore sym wrt origin

graphs of functions:

if a function is symmetric wrt the y-axis,
it is called an even function

if a function is symmetric wrt the origin,
it is called an odd function

so, is $f(x)$ even or odd? how can we tell?

find $f(-x)$ and compare to $f(x)$:

- if $f(-x) = f(x)$, then even
 $f(-x) = -f(x)$, then odd
 $f(-x) =$ neither, then neither even
nor odd

example: if $f(x) = \frac{2}{x^2}$ even, odd, or neither?

$$f(-x) = \frac{2}{(-x)^2} = \frac{2}{x^2} = f(x)$$



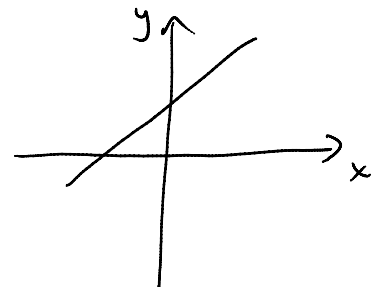
so $f(-x) = f(x)$ \therefore even

if $f(x) = x+1$ even, odd, or neither?

$$f(-x) = -x+1$$

\uparrow
neither $f(x)$ or $-f(x)$

\therefore neither



if $f(x) = x\sqrt{1-x^2}$ even, odd, or neither?

$$f(-x) = -x\sqrt{1-(-x)^2}$$

$$= -x\sqrt{1-x^2}$$

$$= -f(x)$$

-- odd