Section 5.6: cont'd

Newton's Law of Cooling:

A batch of doughnuts is deep-fried at 175°C and then taken out and allowed to cool on the counter. When the room temperature is 25°C, the doughnuts take 5 minutes to cool to 100°C. If Homer can only wait 10 minutes before eating a doughnut and he'll burn his mouth if the doughnuts are hotter than 50°C will he be able to munch on doughnuts happily ever after?

$$T = T_0 + (T_1 - T_0) e^{-kt}$$

find k first:

T_1	T	t
175°C	$\to$ 100°C	in 5 min

with room at 25°C
T_0

$$100 = 25 + (175 - 25) e^{-k \cdot 5}$$

$$75 = 150 e^{-5k} \qquad \Leftarrow \text{divide by 150}$$

$$\frac{1}{2} = e^{-5k}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln e^{-5k}$$

$$\ln \tfrac{1}{2} = -5k$$

$$k = \frac{\ln \tfrac{1}{2}}{-5} \approx 0.138629$$

find T at 10 min:

(could also find t for which $T = 50°C$)

$$T = T_0 + (T_1 - T_0) e^{-kt}$$

$$= 25 + (175 - 25)\, e^{-0.138629 \cdot 10}$$

$$= 62.5°C$$

D'oh! Homer will burn his math.