Section 7.2: cont'd

Tuesday, February 23, 2016
9:33 AM

yesterday, we showed that

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

there are six of these identities, called the **cofunction** identities, as listed on your sheet

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use the sum/difference identities to simplify:

$$\sin\left(x - \pi/2\right) = \sin x \cos\frac{\pi}{2}^{0} - \cos x \sin\frac{\pi}{2}^{1}$$

$$= -\cos x$$

$$\cot(x - 90^\circ) = \frac{\cos(x-90^\circ)}{\sin(x-90^\circ)}$$

$$\cos(x-90^\circ) = \cos x \cos 90^{\circ\,0} + \sin x \sin 90^{\circ\,1}$$

$$= \sin x$$

$$\sin(x-90^\circ) = -\cos x$$ (we just did this one)

so $$\cot(x-90^\circ) = \frac{\sin x}{-\cos x}$$

$$= -\tan x$$

Double-Angle Identities:

how do we get them?

$$\sin 2\theta = \sin(\theta + \theta)$$

$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$\cos 2\theta = \cos(\theta + \theta)$$

$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$\boxed{= \cos^2\theta - \sin^2\theta}$$

$$= \cos^2\theta - (1 - \cos^2\theta)$$

$$\boxed{= 2\cos^2\theta - 1}$$

$$= \cos^2\theta - \sin^2\theta$$

$$= (1 - \sin^2\theta) - \sin^2\theta$$

$$\boxed{= 1 - 2\sin^2\theta}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

example: if $\cos\theta = \frac{1}{3}$, calculate $\cos 2\theta$

method #1:

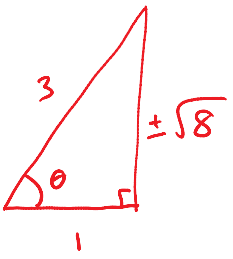
$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= 2\left(\frac{1}{3}\right)^2 - 1$$

$$= \frac{2}{9} - \frac{9}{9}$$

$$= \frac{-7}{9}$$

method #2: $\cos 2\theta = \cos^2\theta - \sin^2\theta$



$$= \left(\frac{1}{3}\right)^2 - \left(\frac{\pm\sqrt{8}}{3}\right)^2$$

$$= \frac{1}{9} - \frac{8}{9}$$

$$= \frac{-7}{9}$$

method #3: $\cos 2\theta = 1 - 2\sin^2\theta$

$$= 1 - 2\left(\frac{\pm\sqrt{8}}{3}\right)^2$$

$$= 1 - 2\left(\frac{8}{9}\right)$$

$$= \frac{9}{9} - \frac{16}{9}$$

$\frac{9}{9} - \frac{16}{9}$

$\frac{-7}{9}$

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simplify:

$$(\sin A + \cos A)^2 - \sin 2A$$

$\sin^2 A + 2\sin A \cos A + \cos^2 A - \sin 2A$

$\sin^2 A + 2\sin A \cos A + \cos^2 A - 2\sin A \cos A$

$\sin^2 A + \cos^2 A$

1

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simplify: $\frac{\sin 2x}{2\cos x}$

$\frac{2 \sin x \cos x}{2 \cos x}$

$\sin x$

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simplify: $\cos^4 x - \sin^4 x$

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$ (1)

$\cos 2x$

$x^4 - y^4 = (x^2+y^2)(x^2-y^2)$

simplify: (nasty!)

$$4\sin x\cos^3 x - 4\sin^3 x\cos x$$

$$4\sin x\cos x\left(\cos^2 x - \sin^2 x\right)$$

$$2\cdot 2\sin x\cos x\left(\cos^2 x - \sin^2 x\right)$$

$$2 \quad \sin 2x \quad \cos 2x \quad \rightarrow$$

$$\sin 2(2x)$$

$$\sin 4x$$

let $\theta = 2x$

$2\sin\theta\cos\theta$

$\sin 2\theta$

$\sin 2(2x)$

$\sin 4x$