

Section 11.7: The Binomial Theorem

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10:58 AM

suppose you were asked to expand $(x+y)^8$

- the hard way: FOIL it all
- the easy way: technology
- the way we're going to do it
Pascal's triangle

$$(x+y)^8 = x^8 + \underline{8}x^7y + \underline{28}x^6y^2 + \underline{56}x^5y^3 + \underline{70}x^4y^4 + \underline{56}x^3y^5 \\ + \underline{28}x^2y^6 + \underline{8}xy^7 + y^8$$

Pascal's triangle:

$$\begin{array}{cccccccc} & & & & & & & 1 \\ & & & & & & 1 & 1 \\ & & & & & 1 & 2 & 1 \\ & & & & 1 & 3 & 3 & 1 \\ & & & 1 & 4 & 6 & 4 & 1 \\ & & 1 & 5 & 10 & 10 & 5 & 1 \\ & 1 & 6 & 15 & 20 & 15 & 6 & 1 \\ 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 \\ 1 & \underline{8} & 28 & 56 & 70 & 56 & 28 & 8 & 1 \end{array}$$

$$(m+2)^5 = m^5 + \underline{5}m^4(2) + \underline{10}m^3 \cdot 2^2 + \underline{10}m^2 \cdot 2^3 + \underline{5}m \cdot 2^4 + 2^5 \\ = m^5 + 10m^4 + 40m^3 + 80m^2 + 80m + 32$$

$$(m-2)^5 = m^5 - 10m^4 + 40m^3 - 80m^2 + 80m - 32$$

↑ if it's a difference, then terms alternate in sign

$$\begin{aligned}(3p-1)^4 &= (3p)^4 - 4(3p)^3 \cdot 1 + 6(3p)^2 \cdot 1^2 - 4(3p) \cdot 1^3 + 1^4 \\ &= 81p^4 - 108p^3 + 54p^2 - 12p + 1\end{aligned}$$