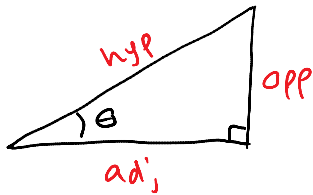


Section 6.1: cont'd

Friday, January 09, 2015
12:29 PM

three basic trig ratios:

(we'll add the other three later on)



SOHCAHTOA

sine

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

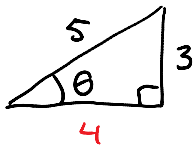
cosine

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

tangent

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

example: find the three basic trig functions of θ for the following triangle:



how do you find remaining side?
Pythagoras or 3-4-5 triangle

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

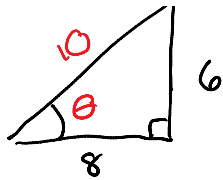
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

what about:



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5}$$



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5}$$

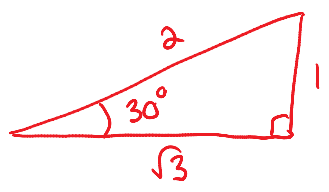
$$\cos \theta = \frac{8}{10} = \frac{4}{5}$$

$$\tan \theta = \frac{6}{8} = \frac{3}{4}$$

same as before!

the trig function of θ depends only on the size of the angle θ and not on the size / scale of the triangle

example: calculate the three basic trig functions of 30° exactly.



$$\sin 30^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

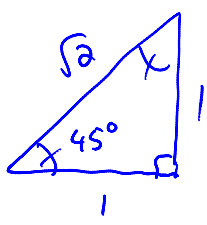
what about 60° ?

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

calculate three basic trig functions of 45° exactly



$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\text{opp}}{\text{adj}} = 1$$

remaining three trig functions of θ : (reciprocal ones)

cosecant

$$\csc \theta = \frac{1}{\sin \theta}$$

secant

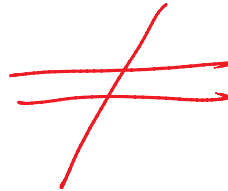
$$\sec \theta = \frac{1}{\cos \theta}$$

cotangent

$$\cot \theta = \frac{1}{\tan \theta}$$

note:

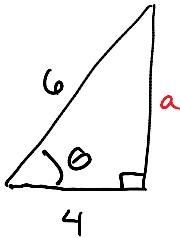
$$\frac{1}{\sin \theta}$$



$$\sin^{-1} \theta$$

(BLAME MATHEMATICIANS)

example: find all six trig functions of θ for the following triangle. Give exact answers.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 &= c^2 - b^2 \\ &= 36 - 16 \\ &= 20 \\ a &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

$$\csc \theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{6} = \frac{2}{3}$$

$$\sec \theta = \frac{3}{2}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

$$\cot \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

calculator exercise:

calculate to 4 decimal places:

$$\sin 35^\circ = 0.5736$$

$$\cos 82^\circ = 0.1392$$

$$\tan 12.5^\circ = 0.2217$$

$$\csc 21.47^\circ = \frac{1}{\sin 21.47^\circ} = 2.7321$$

$$\sec 7.3^\circ = 1.0082$$

$$\cot 69.71125^\circ = 0.3697$$

DMS notation:

two ways of specifying non-integer angles:

decimal: 15.5°

DMS: $15^\circ 30' 00''$
degrees minutes seconds

where 1 degree = 60 minutes
1 minute = 60 seconds
(like time)

conversion:

convert $21^\circ 37' 05''$ to decimal

$$21^\circ 37' 05'' = 21^\circ + 37' + 05''$$

$$= 21^\circ + 37' \left(\frac{1^\circ}{60'} \right) + 05'' \left(\frac{1^\circ}{3600''} \right)$$

$$= 21.6181^\circ$$

convert 52.5437° to DMS

$$52.5437^\circ = 52^\circ + 0.5437^\circ$$

$$= 52^\circ + 0.5437^\circ \left(\frac{60'}{1^\circ} \right)$$

$$= 52^\circ + 32.622'$$

$$= 52^\circ + 32' + 0.622' \left(\frac{60''}{1'} \right)$$

$$= 52^\circ + 32' + 37.32''$$

$$= 52^\circ 32' 37''$$

So we've input the angle to get the trig ratio.
How can we do the reverse: give the trig ratio to find the angle?

example: Calculate θ if $\sin \theta = 0.23$ and θ is acute.

$$\sin \theta = 0.23$$

$$\theta = \sin^{-1} 0.23$$

$$= 13.2971^\circ$$



don't forget the
degrees!

example: calculate θ if θ is acute and

a) $\cos \theta = 0.85$

$$\theta = \cos^{-1}(0.85) = 31.79^\circ$$

b) $\tan \theta = 3$

$$\theta = \tan^{-1}(3) = 71.57^\circ$$

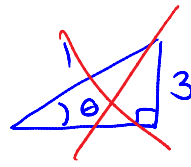
c) $\sin \theta = 3$

undefined

Round to two decimal places.

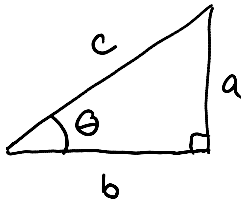
if $\sin \theta = 3$, then

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{1}$$



does not exist

cofunctions and complements:



$$\sin \theta = \frac{a}{c}$$

$$\sin (90^\circ - \theta) = \frac{b}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\cos (90^\circ - \theta) = \frac{a}{c}$$

$$\tan \theta = \frac{a}{b}$$

$$\tan (90^\circ - \theta) = \frac{b}{a}$$

reciprocal!

don't
memorize!

$$\text{So } \sin \theta = \cos (90^\circ - \theta)$$