Section 2.2: The Algebra of Functions

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12:32 PM

consider

$f(x) = x - 2$ domain: $\mathbb{R}$

$g(x) = \frac{1}{x}$ domain: $\{x \mid x \neq 0\}$
$(-\infty, 0) \cup (0, \infty)$

algebra of functions:

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$(f/g)(x) = \frac{f(x)}{g(x)}$$, provided that $g(x) \neq 0$

so, using f & g as defined above, find:

$$(f+g)(x) = f(x) + g(x) = x - 2 + 1/x$$

$$(f-g)(x) = f(x) - g(x) = x - 2 - 1/x$$

$$(fg)(x) = f(x) \cdot g(x) = (x-2)(1/x) = \frac{x-2}{x}$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x-2}{1/x} = x(x-2)$$

what's the domain of $(f+g)(x)$? $\{x \mid x \neq 0\}$

note: this ↗ is the intersection of the domains of f & g

$(f-g)(x)$? same

$(fg)(x)$? same

$(f/g)(x)$? same

even though $(f/g)(x) = \underbrace{x(x-2)}_{\text{no domain problems,}}$

still have to have $g(x)$ existing so restriction is still $\{x \mid x \neq 0\}$

what is $(g/f)(x)$?

$$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{1/x}{x-2} = \frac{1}{x} \cdot \frac{1}{x-2}$$

$$= \frac{1}{x(x-2)}$$

domain : $f: \mathbb{R}$ $g: \{x \mid x \neq 0\}$ and recall that we also can't have the denominator zero

$\{x \mid x \neq 0$ and $x \neq 2\}$

$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

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example: let $f(x) = \sqrt{x+6}$ domain: $\{x \mid x \geq -6\}$

$g(x) = \frac{1}{x}$ domain: $\{x \mid x \neq 0\}$

find $(f-g)(x)$ and its domain
$(fg)(x)$ " " "
$(f/g)(x)$ " " "
$(g/f)(x)$ " " "

$(f-g)(x) = f(x) - g(x) = \sqrt{x+6} - \frac{1}{x}$ so $x \geq -6$ and $x \neq 0$

$\{x \mid x \geq -6 \text{ and } x \neq 0\}$

$[-6, 0) \cup (0, \infty)$

$(fg)(x) = \frac{\sqrt{x+6}}{x}$ same

$(f/g)(x) = \frac{\sqrt{x+6}}{1/x} = x\sqrt{x+6}$ same

$(g/f)(x) = \frac{1/x}{\sqrt{x+6}} = \frac{1}{x\sqrt{x+6}}$ $\left(= \frac{\sqrt{x+6}}{x(x+6)} \right)$

domain: see below

note: in Math 173: if the radical in the denominator CONTAINS VARIABLES, no need to rationalize

note: for $f(x) = \sqrt{x+6}$, can do this

$x + 6 \geq 0$

$x \geq -6$

for the domain of $\frac{g(x)}{f(x)} = \frac{1}{x\sqrt{x+6}}$, will have the intersection of domains of f & g as usual, plus the fact that $f(x) \neq 0$

$$\underbrace{f(x) \neq 0}_{\sqrt{x+6} \neq 0}$$

$\sqrt{x+6} \neq 0$ so $x = -6$ no longer works

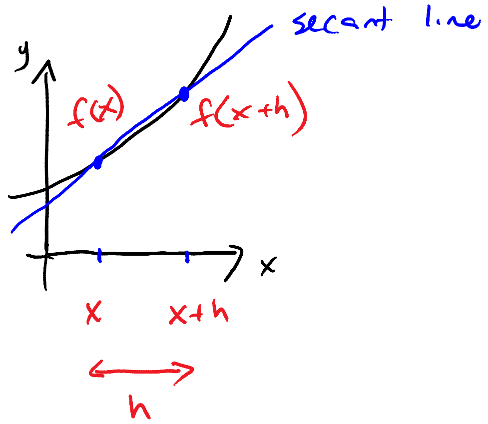
$$(-6, 0) \cup (0, \infty)$$

or $\{ x \mid x > -6 \text{ and } x \neq 0 \}$

difference quotient:

$$\frac{f(x+h) - f(x)}{h}$$

digression: why do we care?



slope of secant line

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

example: find the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

for $f(x) = 3x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \frac{6xh + 3h^2}{h}$$

$$= \frac{\cancel{h}(6x+3h)}{\cancel{h}} = 6x + 3h$$