

# Section 2.3: Composition of Functions

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12:57 PM

Composition of functions:

the composite function  $f \circ g$

- "f circle g"
- "f composed with g"
- "the composition of f and g"

$$(f \circ g)(x) = f(g(x))$$

example:

$$\text{let } f(x) = \sqrt{x}$$

$$g(x) = x+3$$

$$\text{domain: } \{x \mid x \geq 0\}$$

$$\text{domain: } \mathbb{R}$$

then find  $(f \circ g)(x)$  and  $(g \circ f)(x)$   
and their domains

$$(f \circ g)(x) = f(g(x))$$

$$\begin{aligned} (f \circ g)(x) &= \sqrt{g(x)} \\ &= \sqrt{x+3} \end{aligned}$$

$$\begin{aligned} (f \circ g)(x) &= f(x+3) \\ &= \sqrt{x+3} \end{aligned}$$

$$\text{domain: } \{x \mid x \geq -3\}$$

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$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= \sqrt{x} + 3\end{aligned}$$

$$\text{domain: } \{x \mid x \geq 0\}$$

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example: let  $f(x) = \frac{1}{x-2}$  and  $g(x) = \sqrt{x}$

find  $(f \circ g)(x)$  and  $(g \circ f)(x)$  and their domains

$$\begin{aligned}(f \circ g)(x) &= f(\sqrt{x}) \\ &= \frac{1}{\sqrt{x}-2}\end{aligned}$$

domain:

$$\{x \mid x \neq 4 \text{ and } x \geq 0\}$$

$$[0, 4) \cup (4, \infty)$$

$$\begin{aligned}(g \circ f)(x) &= \sqrt{\frac{1}{x-2}} \\ &= \frac{1}{\sqrt{x-2}}\end{aligned}$$

domain:

$$\{x \mid x > 2\}$$

$$(2, \infty)$$

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decomposition of functions:

$$\text{let } h(x) = (f \circ g)(x) = f(g(x))$$

if  $h(x) = \sqrt{2x^2 + 5}$ , then list possible sets of  $f$  to  $g$  that make this work

$$\begin{cases} f(x) = \sqrt{x} \\ g(x) = 2x^2 + 5 \end{cases}$$

$$\begin{cases} f(x) = \sqrt{x + 5} \\ g(x) = 2x^2 \end{cases}$$

$$\begin{cases} f(x) = \sqrt{2x + 5} \\ g(x) = x^2 \end{cases}$$

$$\begin{cases} f(x) = \sqrt{2(x - 157)^2 + 5} \\ g(x) = x + 157 \end{cases}$$