Section 2.4: cont'd

Tuesday, January 20, 2015
11:35 AM

algebraic tests for symmetry:

- if replacing y by $-y$ gives an equivalent equation, symmetric wrt x-axis
- if replacing x by $-x$ gives an equivalent equation, symmetric wrt y-axis
- if replacing x by $-x$ and y by $-y$ gives an equivalent equation, symmetric wrt origin

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example: Use the algebraic test for symmetry on the relation $x = y^2$ to determine the symmetry.

x-axis: replace y with $-y$

$$x = y^2$$
$$x = (-y)^2 \quad \Leftarrow \text{replace } y \text{ with } -y$$
$$x = y^2$$

same!

symmetric wrt x-axis

y-axis: replace x by $-x$

$$x = y^2$$
$$-x = y^2 \quad \Leftarrow \text{replace } x \text{ by } -x$$

not equivalent!

not symmetric wrt y-axis

origin : replace x by $-x$ and y by $-y$

$$x = y^2$$
$$-x = (-y)^2 \quad \text{after replacement}$$
$$-x = y^2$$

not equivalent

$\therefore$ not symmetric wrt origin

note: $y = x^3$ $\Leftarrow$ do origin test

$$-y = (-x)^3$$
$$-y = -x^3$$
$$y = x^3$$

same

$\therefore$ symmetric wrt origin

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graphs of functions:

if a function is symmetric wrt the y-axis, it is called an even function

if a function is symmetric wrt the origin, it is called an odd function

so, is $f(x)$ even or odd? how can we tell?

find $f(-x)$ and compare to $f(x)$:

if $f(-x) = f(x)$, then even
$f(-x) = -f(x)$, then odd
$f(-x) =$ neither, then neither even nor odd

example: is $f(x) = \frac{2}{x^2}$ even, odd, or neither?

$f(-x) = \frac{2}{(-x)^2} = \frac{2}{x^2} = f(x)$

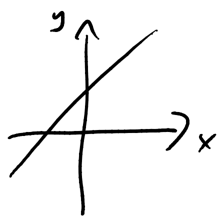
$f(-x) = f(x)$ $\therefore$ even

is $f(x) = x+1$ even, odd, or neither?

$-f(x) = -x-1$

$f(-x) = -x+1$ neither

note:



is $f(x) = x\sqrt{1-x^2}$ even, odd, or neither?

$f(-x) = -x\sqrt{1-(-x)^2}$

$= -x\sqrt{1-x^2}$

$= -f(x) \qquad \therefore \text{ odd}$