Section 4.1 and 4.2: cont'd

Tuesday, January 27, 2015
11:29 AM

example: For the following polynomial, find the zeros and their multiplicities, then sketch the graph.

$$P(x) = -x^3 + 2x^2 + 4x - 8$$

$\leftarrow$ y-int

$$= -(x^3 - 2x^2 - 4x + 8)$$

$$= -[x^2(x-2) - 4(x-2)]$$

$$= -[(x-2)(x^2-4)]$$

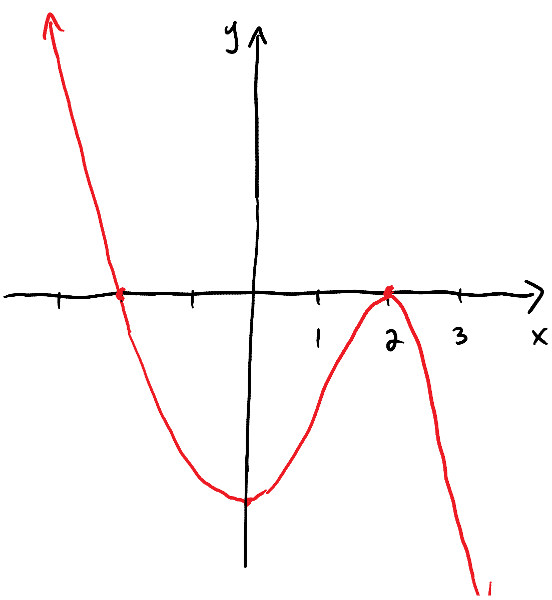
$$= -(x-2)(x+2)(x-2)$$

$$= -(x-2)^2(x+2)$$

zeros: $x = 2, -2$

2: mult 2

-2: mult 1



end behavior: $\uparrow$ $\downarrow$ (lc is neg, degree is odd)

y-int: $P(0) = -8$

so $(0,-8)$ is

y-intercept

note: if you need a couple of extra points just plug in some values of x into $P(x)$

$\rightarrow$ for this graph, $x = -1, 1$ and maybe $3, -3$ good choices

example: Find the zeros and their associated multiplicities for:

a) $P(x) = x^4 - 9x^2 + 20$

$= x^4 - 4x^2 - 5x^2 + 20$

$= x^2(x^2 - 4) - 5(x^2 - 4)$

$= (x^2 - 5)(x^2 - 4)$

$= (x - \sqrt{5})(x + \sqrt{5})(x - 2)(x + 2)$

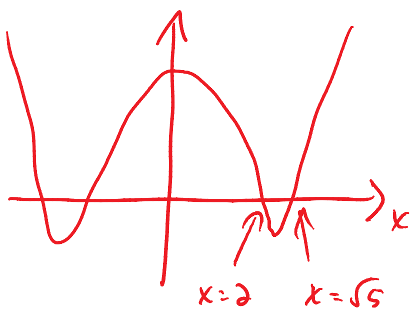
$ac = 20$

1	20
2	10
-4	-5

$x = \sqrt{5}, -\sqrt{5}, 2, -2$

mult 1, mult 1, mult 1, mult 1

note: quick sketch



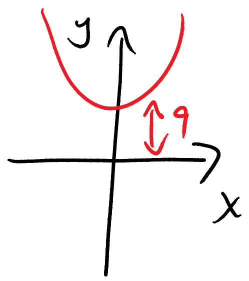
b) $Q(x) = x^2 + 9$

$= x^2 - (-9)$

$= (x - 3i)(x + 3i)$

$x = 3i, \; -3i$ each with mult 1

note:



note: $Q(x) = x^2 + 9$

$0 = x^2 + 9$

$x^2 = -9$

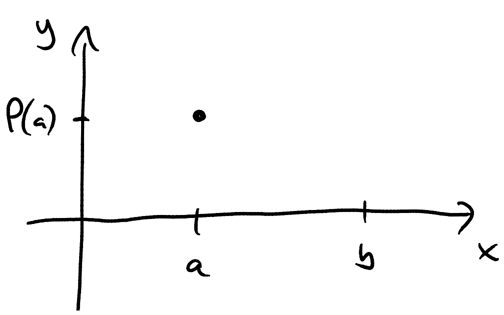
$x = \pm\sqrt{-9} = \pm\sqrt{9}\sqrt{-1}$

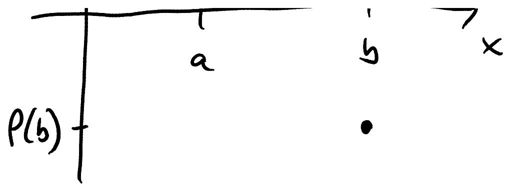
$= \pm 3i$

Intermediate Value Theorem

- for any polynomial $P(x)$ with real coefficients suppose that for $a \neq b$, $P(a)$ and $P(b)$ have opposite signs

$\rightarrow$ then $P(x)$ must have a real zero between a & b




a
b
x
f(b)