Section 4.4: Theorems about Zeros of Polynomial Functions

Wednesday, January 28, 2015
1:49 PM

Fundamental Theorem of Algebra:

every polynomial of degree n for $n \geq 1$ has at least one complex zero

real numbers are a subset of complex

→ but this theorem doesn't actually tell us how to find it/them

(thanks, guys!)

why do we care?

every polynomial of degree $n \geq 1$ can be written as

n zeros

$$f(x) = a_n (x - c_1)(x - c_2)(x - c_3) \dots (x - c_n)$$

leading coefficient

example: give a polynomial of degree 3 with zeros -1, 0, and 4 (answers may vary)

$$P(x) = 22(x + 1)(x - 0)(x - 4)$$

↳ can use any non-zero real here

$= 22\,x(x+1)(x-4)$

note: if polynomial has real coeffs,

then if $a+bi$ is a zero, then
$a-bi$ is also a zero

$\underbrace{}_{\text{complex conjugates}}$

and if polynomial has rational coeffs,

then if $a+\sqrt{b}$ is a zero,
then $a-\sqrt{b}$ is also a zero

Rational Zeros Theorem: ↙ please remember what this one is called!

consider a polynomial with integer coeffs.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots$$
$$a_2 x^2 + a_1 x + a_0$$

↑ leading coeff

↑ constant term

let p = all possible factors of a_0

q = all possible factors of a_n

then $\frac{p}{q}$ = list of all possible rational zeros of $P(x)$

→ this gives you the list of numbers to check

example: Use the Rational Zeros Theorem to list all possible rational zeros of

$$P(x) = 2x^4 + 3x - 10$$

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2}$$

$$= \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}$$