Section 4.4: cont'd

Thursday, January 29, 2015
11:31 AM

example: factor the following polynomial into linear factors, and then state the zeros and multiplicities

$$P(x) = x^3 + 6x^2 + 12x + 8$$

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1} = \underbrace{\pm 1, \pm 2, \pm 4, \pm 8}$$

note: none of the positives will work

$P(-1) \neq 0$

$P(-2) = 0$ so $(x+2)$ is a factor

$$\begin{array}{r|l} & x^2 + 4x + 4 \\ \hline x+2 & x^3 + 6x^2 + 12x + 8 \\ & \underline{x^3 + 2x^2} \\ & \quad 4x^2 + 12x \\ & \quad \underline{4x^2 + 8x} \\ & \qquad 4x + 8 \end{array}$$

and $x^2 + 4x + 4 = (x+2)^2$

conclusion: $f(x) = (x+2)^3$

$x = -2$

mult 3

recall: synthetic division (OPTIONAL!)

$$\frac{x^3+7x^2+15x+25}{x+5}$$

$$\begin{array}{r|l} & x^2 + 2x + 5 \\ \hline x+5 & x^3+7x^2+15x+25 \\ & x^3+5x^2 \\ \hline & \quad 2x^2+15x \\ & \quad 2x^2+10x \\ \hline & \qquad 5x+25 \end{array}$$

$$\begin{array}{r|rrrr} -5 & 1 & 7 & 15 & 25 \\ & & -5 & -10 & -25 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$\underbrace{1 \quad 2 \quad 5}_{x^2+2x+5}$; 0: remainder

example: factor into linear factors

$$P(x) = x^3+3x^2+x-5$$

$\frac{p}{q} = \pm 1, \pm 5$

$P(1) = 0$ so $(x-1)$ is a factor

so $1 \mid 1 \quad 3 \quad 1 \quad -5$

so

$$\begin{array}{c|cccc} 1 & 1 & 3 & 1 & -5 \\ & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & 0 \\ & x^2 & +4x & +5 & \text{rem} \end{array}$$

but x^2+4x+5 doesn't factor easily!

so set it to zero and find the zeros:

$$x^2+4x+5=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-4\pm\sqrt{16-20}}{2}$$

$$=\frac{-4\pm\sqrt{-4}}{2}$$

$$=\frac{-4\pm 2i}{2}$$

$$=-2\pm i$$

which means that

$$x^2+4x+5=(x-(-2+i))(x-(-2-i))$$

$$=(x+2-i)(x+2+i)$$

Conclusion: $P(x)=(x-1)(x+2-i)(x+2+i)$