Section 4.4: cont'd

Friday, January 30, 2015
12:35 PM

Descartes' Rule of Signs

$$f(x) = x^3 + 6x^2 + 12x + 8$$

number of sign changes in coeffs = 0

(max) number of positive zeros is zero

$$f(-x) = (-x)^3 + 6(-x)^2 + 12(-x) + 8$$

$$= -x^3 + 6x^2 - 12x + 8$$

number of sign changes is 3

max number of negative zeros is 3

actual number of negative zeros is either 3 or 1

rule: the maximum number of positive real zeros of $P(x)$

= the number of sign changes in coeffs of $P(x)$

→ then, can count down by twos to get the possible number(s) of positive real zeros

example: suppose you find 6 sign changes in coeffs of $P(x)$

max of 6 positive real zeros

can have 6 or 4 or 2 or 0

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5 changes:

either have 5 or 3 or 1 positive real zeros (at least one)

The maximum number of negative real zeros

= number of sign changes in coeffs of $P(-x)$

example: use Descartes' rule to determine how many positive and negative real zeros the following polynomial could have:

$$P(x): 2x^7 - x^6 - x^5 + x^4 - x^3 + x - 1$$

positive real zeros: 5 or 3 or 1

$$P(-x): -2x^7 - x^6 + x^5 + x^4 + x^3 - x - 1$$

$P(-x) = -2x^7 - x^6 + x^5 + x^4 + x^3 - x - 1$

negative real zeros: 2 or 0