Section 4.5: Rational Functions

Friday, January 30, 2015
1:09 PM

rational function $\equiv$ a function which is a quotient of polynomials

$f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials

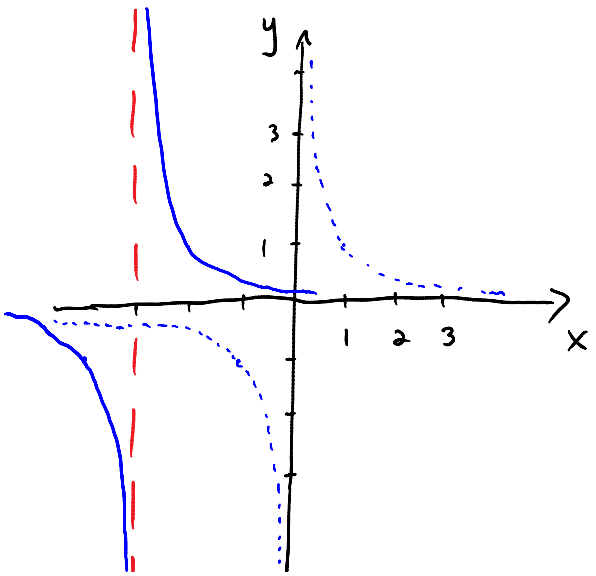
domain of $f(x)$: all x such that $Q(x) \neq 0$

example: state the domain of

$$f(x) = \frac{1}{x+3}$$

domain: $\{x \mid x \neq -3\}$

note: this is just the graph of $f(x) = \frac{1}{x}$ shifted to the left by 3



↑ the vertical line at $x = -3$ is called a

vertical asymptote

- shows x-values not in the domain
- graph will **never** cross vertical asymptotes

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horizontal asymptotes: $f(x) = \frac{P(x)}{Q(x)}$

① degree of $P(x)$ < degree of $Q(x)$

examples: $f(x) = \frac{1}{x+3}$, $f(x) = \frac{5x^3}{2x^3-1}$

→ will have horizontal asymptote at

$$\boxed{y = 0}$$

② degree of $P(x)$ = degree of $Q(x)$

examples: $f(x) = \frac{5x^2-2}{15x^2+4x}$, $f(x) = \frac{9x+1}{x+3}$

↑ ↑

asymptote $y = 1/3$ $y = 9$

the horizontal asymptote is just the

ratio of leading coefficients

③ degree of $P(x)$ > degree of $Q(x)$

no horizontal asymptotes

(slant/oblique → we will do later)

note: horizontal asymptotes only deal with behaviour as

$x \to \infty$ and $x \to -\infty$

(the "far reaches" of the graph / end behaviour)

∴ horizontal asymptotes have nothing to say about the exciting bits in the "middle" of the graph

⇒ graph can (but does not have to)

cross horizontal asymptotes in middle of graph

And then we did the handout, examples 1 and 2.