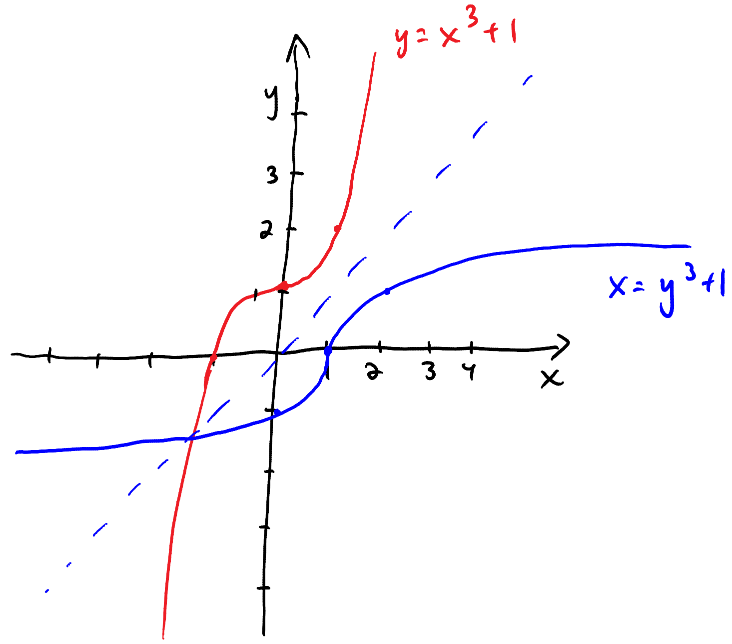
# Section 5.1: cont'd

Tuesday, February 03, 2015
11:23 AM

example: graph $y = x^3 + 1$ and its inverse

| $x$ | $y = x^3+1$ |
|---|---|
| -2 | -7 |
| -1 | 0 |
| 0 | 1 |
| 1 | 2 |
| 2 | 9 |




how to do inverse?

1. take previous table and swap x and y
2. just mirror about the line $y = x$
3. swap x and y in equation:

$$y = x^3 + 1 \implies x = y^3 + 1$$

and solve for y

$$x - 1 = y^3$$

$$y = \sqrt[3]{x-1}$$

one-to-one functions:

for $a \neq b$, then $f(a) \neq f(b)$

→ different x-values give different y-values

example: is $f(x) = x^2$ a one-to-one function?

$f(2) = 4$
$f(-2) = 4$

NO!

properties:

- if the function is one-to-one, its inverse is ~~is~~ also a function
- domain of function is the range of the inverse
  range " " " " domain " " "

note: a function that only increases over the domain (or only decreases) is one-to-one

how can you tell from the graph of a function that the inverse will also be a function?

horizontal line test - if passes, inverse is a function

---

notation for inverses:

the inverse of $f(x)$ is written $f^{-1}(x)$

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

obtaining formulae for inverses: $f(x) = x^3+1$

① replace $f(x)$ with $y$ REPLACE $y = x^3+1$

② interchange $x$ and $y$ SWAP $x = y^3+1$

③ solve for $y$ SOLVE $y = \sqrt[3]{x-1}$

④ replace $y$ by $f^{-1}(x)$ REPLACE $f^{-1}(x) = \sqrt[3]{x-1}$

example: find inverse of $f(x) = \dfrac{3}{x+2}$

$y = \dfrac{3}{x+2}$ REPLACE

$x = \dfrac{3}{y+2}$ SWAP

$x(y+2) = 3$ SOLVE

$y+2 = \dfrac{3}{x}$

$$y = \frac{3}{x} - 2$$

$$f^{-1}(x) = \frac{3}{x} - 2 \qquad \text{REPLACE}$$

$$\left( \text{or} \quad \frac{3-2x}{x} \right)$$

domain of $f(x)$: $\{x \mid x \neq -2\}$

range of $f(x)$: $\{☺ \mid ☺ \neq 0\}$

domain of $f^{-1}(x)$: $\{x \mid x \neq 0\}$

range of $f^{-1}(x)$: $\{☹ \mid ☹ \neq -2\}$

---

inverse functions and <u>composition</u>:

how to tell if two functions are inverses of each other:

$$\left. \begin{aligned} (f^{-1} \circ f)(x) &= f^{-1}(f(x)) = x \\ (f \circ f^{-1})(x) &= f(f^{-1}(x)) = x \end{aligned} \right\} \text{must test both}$$