Section 5.3: Logarithmic Functions and Graphs

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12:32 PM

here's the basic idea behind the log function:

solve for x: $y = x^3 \Rightarrow \sqrt[3]{y} = x \Rightarrow x = \sqrt[3]{y}$

$y = 3^x \Rightarrow \log_3 y = x \qquad x = \log_3 y$

the log function allows me to solve for the exponent when the base is known

logs can be used to solve equations like

$$10^x = 7$$

$$x = \log_{10} 7$$

$$\approx 0.845$$

how do these logs work?

$\log_2 16$ ← what exponent on 2 (the base) gives 16 (the argument)?

argument ↙ ↑ base

$$\log_2 16 = 4$$

example: simplify:

$\log_2 8 = 3$

$\log_2 \frac{1}{2} = \log_2 2^{-1} = -1$

$\log_2 \sqrt[3]{2} = \log_2 2^{1/3} = \frac{1}{3}$

$\log_2 64 = \log_2 2^6 = 6$

$\log_8 2 = \log_8 8^{1/3} = \frac{1}{3}$

$\log_2 1 = 0$

$\log_2(-2) =$ undefined

$\log_2 0 =$ undefined

$\log 1000 = 3$

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definition:

$y = \log_a x$ is equivalent to $a^y = x$

for $a > 0$ and $a \neq 1$
and $x > 0$

rewrite the following in either log or exponential form:

$10^x = 7$ $\qquad$ $z = a^{y+1}$ $\qquad$ $\log_3 m = 5$

$x = \log_{10} 7$ $\qquad$ $\log_a z = y+1$ $\qquad$ $m = 3^5$

notation:

common log $\qquad$ natural log

$\log_{10} x = \log x$ $\qquad$ $\log_e x = \ln x$

calculator check: $\qquad$ round to 3 decimals

$\ln 7 = 1.946$

$\log 35 = 1.544$

how about $\log_3 5$?

use base-change formula:

$$\log_3 5 = \frac{\log 5}{\log 3} = \frac{\ln 5}{\ln 3} \approx 1.465$$

$\log 3$

in general:

$$\log_b M = \frac{\log_a M}{\log_a b} \left[= \frac{\log M}{\log b} = \frac{\ln M}{\ln b} \right]$$

now calculate: (round to 3 decimals)

$\log_2 17 \approx 4.087$

$\log_{1.02} 5 \approx 81.274$

$\log_{157} 0.1 \approx -0.455$

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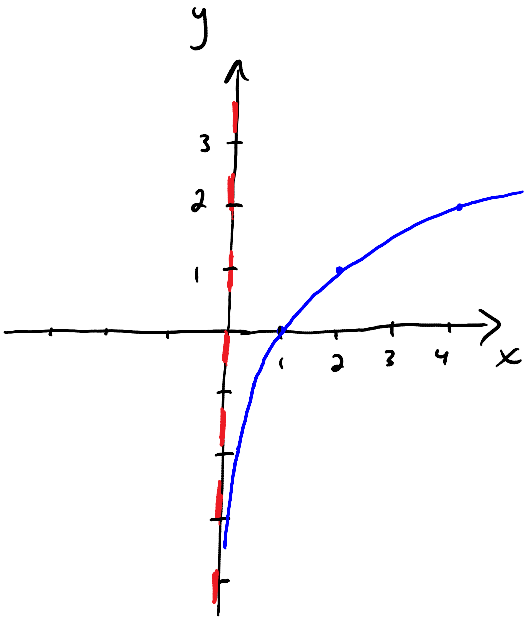
sketch $y = \log_2 x$

equivalent to $2^y = x$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2



↑ vertical asymptote

domain : $\{x \mid x > 0\}$ $(0, \infty)$

range : R

graph $y = \log_2(x+3)$ is just ↑ shifted left by 3

$y = \log_2 x + 3$ is just ↑ shifted up by 3