Section 5.4: cont'd:

Wednesday, February 11, 2015
12:35 PM

BAD MATH:

fill in the blank with $=$ or $\neq$ to make the statement true

$$\log_a MN \ \underline{\neq}\ (\log_a M)(\log_a N)$$

$$\log_a (M+N) \ \underline{\neq}\ \log_a M + \log_a N$$

$$\log_a \left(\frac{M}{N}\right) \ \underline{\neq}\ \frac{\log_a M}{\log_a N}$$

$$\frac{\log_a M}{\log_a N} \ \underline{\neq}\ \frac{M}{N}$$

base-change $\rightarrow$ $\dfrac{\log M}{\log N} \ \underline{=}\ \dfrac{\ln M}{\ln N} = \log_N M$

$$(\log_a M)^p \ \underline{\neq}\ p \log_a M$$

recall: $\log_a (M^p) = p \log_a M$

example:

express as a single log, if possible, and simplify if appropriate

① $\log (x+2) + \log (x-2) - 5 \log x$

$\log (x+2)(x-2) - \log x^5$

$\log \dfrac{(x+2)(x-2)}{x^5}$ or $\log \dfrac{x^2-4}{x^5}$

② $-\frac{1}{3}\ln 8 + \frac{1}{2}\ln 16$

$\ln \dfrac{1}{\sqrt[3]{8}} + \ln\sqrt{16}$

$\ln \frac{1}{2} + \ln 4$

$\ln 2$

$-\frac{1}{3}\ln 2^3 + \frac{1}{2}\ln 2^4$

$\ln 2^{-1} + \ln 2^2$

$-\ln 2 + 2\ln 2$

$\ln 2$

③ $\log x + \ln y$ not same base

$\log x + \ln y$

---

example: given that $\log_a x = 2$ and $\log_a y = 3$, evaluate

① $\log_a(xy) = \log_a x + \log_a y = 2+3 = 5$

② $\log_a\left(\dfrac{x^2}{y^3}\right) = 2\log_a x - 3\log_a y = 4-9 = -5$

③ $\log_a\left(\dfrac{\sqrt{x}}{a}\right) = \frac{1}{2}\log_a x - \log_a a = 1-1 = 0$

④ $\dfrac{\log_a x}{\log_a y} = \dfrac{2}{3}$

note: $\log_a(x+y)$ cannot be

simplified using these log properties!

further rules:

$\log_a a^x = x$ and $a^{\log_a x} = x$

$\Downarrow$

$\therefore \log_a a = 1$

and $\log_a 1 = 0$

examples: simplify:

$\log_x x^4 = 4$

$\log_a \sqrt[3]{a} = 1/3$

$17^{\log_{17} y} = y$ $\Leftarrow$ $17^{\log_{17} y} = x$

$\log_{17} y = \log_{17} x$

$y = x$

$\ln e^x = x$

trickier:

$8^{\log_8 3} = 3$

$8^{2\log_8 3} = 8^{\log_8 3^2} = 3^2$ or 9

$8^{\log_2 3} = (2^3)^{\log_2 3} = 2^{3\log_2 3} = 2^{\log_2 3^3} = 3^3$ or 27

$8^{\log_8 2 + \log_8 3} = 8^{\log_8 6} = 6$

or $8^{\log_8 2}\, 8^{\log_8 3} = 2 \cdot 3 = 6$