Section 5.5: Solving Exponential & Log Eqns

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1:21 PM

exponential equations:

$2^{3x+1} = 64$ $\qquad$ $3^x = 20$

$2^{3x+1} = 2^6$

$3x+1 = 6$

$3x = 5$

$x = 5/3$ ← either

$\{5/3\}$ ↙

base-exponent property:

for any $a > 0$ and $a \neq 1$, $\qquad$ (a is a legal base)

$a^x = a^y$ is equivalent to $x = y$

similar property for logs:

$\log_a M = \log_a N$ is equivalent to $M = N$

↖ ↗

must be <u>same</u> base

but what if we can't match bases?

$3^x = 20$

method #1

$3^x = 20$

$x = \log_3 20$

$= \frac{\log 20}{\log 3}$

≈ 2.72683

method #2

$3^x = 20$

$\log 3^x = \log 20$

$x \log 3 = \log 20$

$x = \frac{\log 20}{\log 3}$

example: Give both an exact solution and an approximate answer to three decimals.

$$e^{2x-5} = 18$$

$$\ln e^{2x-5} = \ln 18$$

recall $\ln e^x = x$

$$2x - 5 = \ln 18$$

$$2x = \ln 18 + 5 = 5 + \ln 18$$

$$x = \frac{5 + \ln 18}{2} \approx 3.94519$$

$$\approx 3.945$$

$e^{2x-5} = 18$

$2x - 5 = \log_e 18$

$2x - 5 = \ln 18$

$$10^{1-x} = 0.06$$

$$\log 10^{1-x} = \log 0.06$$

$$(1-x)\log 10 = \log 0.06$$

$$1-x = \log 0.06$$

$$x = 1 - \log 0.06$$
$$\approx 2.22185$$
$$\approx 2.222$$

nasty:

$$2^x = 3^{x+1}$$

$$\ln 2^x = \ln 3^{x+1}$$

$$x \ln 2 = (x+1)\ln 3$$

$$x \ln 2 = x \ln 3 + \ln 3$$

$$x \ln 2 - x \ln 3 = \ln 3$$

$$x(\ln 2 - \ln 3) = \ln 3$$

$$x = \frac{\ln 3}{\ln 2 - \ln 3}$$

$$\approx -2.70951$$
$$\approx -2.710$$

Solving logarithmic equations:

$$\log_5 (x+2) = -1$$

$$x+2 = 5^{-1}$$

$$x+2 = 1/5$$

$$x = -2 + 1/5 = \frac{-10}{5} + \frac{1}{5} = \frac{-9}{5}$$

check: $\log_5 (-9/5 + 2) = -1$

$\log_5 (1/5) = -1$ ✓