Section 5.6: Cont'd

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12:31 PM

According to Monday Magazine, "one rat can create 15,000 descendents in a single year." Assuming this is true

a) calculate the doubling time for Victoria's rat population

b) how long would it take for that one rat to have one million descendents instead?

a) $A = A_0 e^{rt}$

$2A_0 = A_0 e^{rt}$ after one doubling time

$\Leftarrow$ now divide both sides by A_0

$2 = e^{rt}$ $\Leftarrow$ problem! two variables!

but how do we find r?

use that 1 rat $\to$ 15000 in one year

$A = A_0 e^{rt}$

$15000 = 1 e^{r \cdot 1}$

$15000 = e^{r}$

$\ln 15000 = \ln e^{r}$

$\ln 15000 = r$ or $r \approx 9.615805$ *

* don't round too early - keep at least 4-5 sigfigs

now return to doubling time

$$A = A_0 e^{rt}$$
$$2A_0 = A_0 e^{rt}$$
$$2 = e^{rt}$$
$$2 = e^{9.615805t}$$
$$\ln 2 = 9.615805t$$
$$t = \frac{\ln 2}{9.615805}$$
$$= 0.072084 \qquad \times \left(\frac{365 \text{ days}}{1 \text{ year}}\right)$$
$$\approx 0.072 \text{ years}$$

or convert to days

$$t = 26.3 \text{ days}$$

b) $A = A_0 e^{rt}$

$$10^6 = 1\, e^{9.615805t}$$
$$\ln 10^6 = 9.615805t$$
$$t = \frac{\ln 10^6}{9.615805} \approx 1.4 \text{ years}$$

exponential decay:

Scientists at the Kellogg Radiation Lab are studying the radioactive isotope Linoleum-237. The halflife of Ln-237 was found to be 57 days. If initially the lab had 1.5 g of Ln-237, how much was left after exactly 3 weeks had elapsed?

finding r: $A_0 \to \frac{1}{2}A_0$ after 57 days

$$A = A_0 e^{-rt}$$

$$\tfrac{1}{2}A_0 = A_0 e^{-r \cdot 57}$$

$$\tfrac{1}{2} = e^{-57r}$$

$$\ln \tfrac{1}{2} = \ln e^{-57r}$$

$$\ln \tfrac{1}{2} = -57r$$

$$r = \frac{\ln \frac{1}{2}}{-57} \approx 0.0121605$$

now find A:

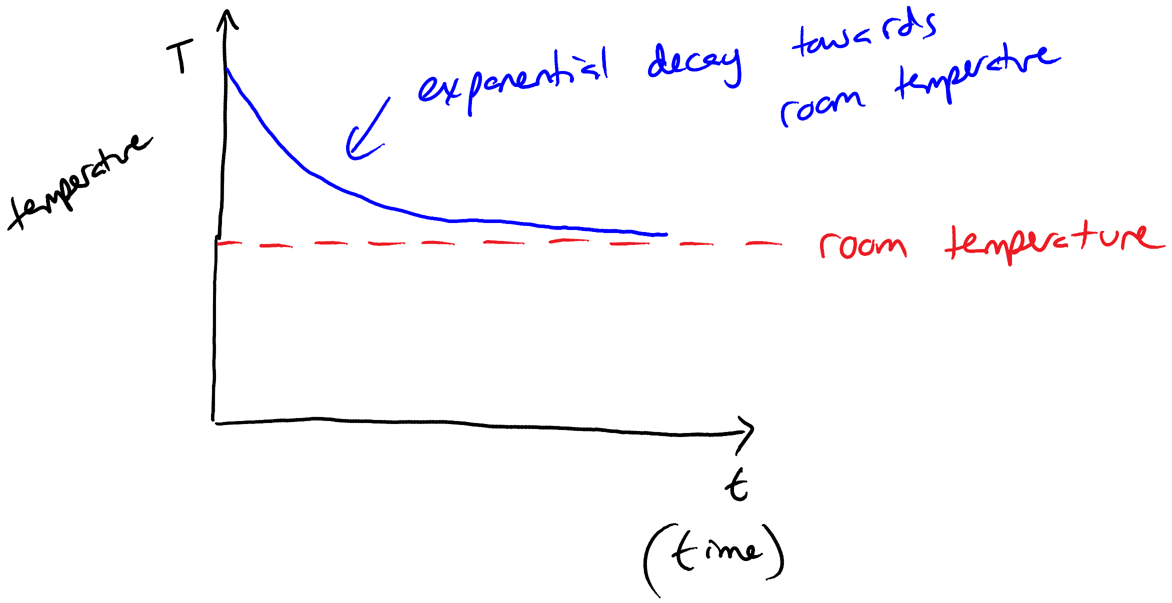
$$A = A_0 e^{-rt}$$

$$= 1.5\, e^{-0.0121605(21)}$$

$$= 1.161944$$

$$= 1.2\text{ g}$$

Newton's Law of Cooling:



$$T = T_0 + (T_1 - T_0)e^{-kt}$$

$T(t)$

function notation:

Temp is a function of time

T : temp of object at time t

T_0 : room temperature

T_1 : initial temp of object

k : rate

t : time elapsed

A batch of doughnuts is deep-fried at 175°C and and then taken out and allowed to cool on the counter. When the room temperature is 25°C, the doughnuts take 5 minutes to cool to 100°C. If Homer can only wait 10 minutes before eating a doughnut and he'll burn his mouth if the doughnut is hotter than 50°C, will he be able to munch doughnuts happily ever after?

$$T = T_0 + (T_1 - T_0)\, e^{-kt}$$

T_0 = room temp
T_1 = initial temp of object

find k: $T_1 = 175°C \rightarrow T = 100°C$ after 5 min
$T_0 = 25°C$

$$100 = 25 + (175 - 25)\, e^{-k \cdot 5}$$

$$75 = 150\, e^{-5k}$$

$$\tfrac{1}{2} = e^{-5k}$$

$$\ln \tfrac{1}{2} = -5k$$

$$k = -\frac{\ln \frac{1}{2}}{5} \approx 0.138629$$

find T at 10 min: (could also find t for which $T = 50°C$)

$$T = T_0 + (T_1 - T_0)\, e^{-kt}$$

$$= 25 + (175 - 25)\, e^{-0.138629(10)}$$

$$= 62.5°C$$

D'oh! Homer will burn his mouth.

note:

$$75 = 150\, e^{-5k}$$

note: better to divide by 150 <u>first</u>

$$\ln 75 = \ln 150\, e^{-5k}$$

$$= \ln 150 + \ln e^{-5k}$$

$$\ln 75 - \ln 150 = -5k \cancel{\ln e}^{\,1}$$

$$\underbrace{\ln 75 - \ln 150}_{\ln \frac{75}{150}} = -5k$$

moral of story: divide by coeff <u>first</u>