Section 6.4: Radians and Arclength

Tuesday, February 17, 2015
11:32 AM

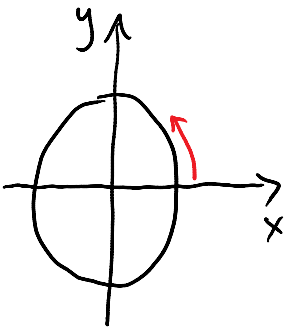
(note: we will omit linear/angular speed)

radian - SI unit for measuring angles

note: degree is a practical unit in SI
(like hours, days for time)

how big is a radian?

consider a unit circle (radius = 1 unit)



starting on the positive x-axis and walking along the unit circle, how far will we have walked if we complete one revolution?

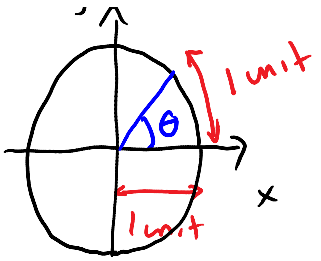
$C = 2\pi r$ ($r = 1$)

$= 2\pi$

what about ½ revolution? π

here's the big idea:

If I walk a distance of exactly one unit along the circle, through what angle have I walked?



θ = 1 radian

≡ angle subtended by an arclength of one unit on the unit circle

so 1 rad is a little less than 60°
3 rads " " " " " 180°

and $\boxed{\pi \text{ rads} = 180^\circ}$

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also note:

$$\theta = \frac{\text{arclength}}{\text{radius}}$$

← units of length

↑ units of length

↗ in rads

so radians are a dimensionless quantity

(and you can leave the "radians" off your angle and still be correct)

Unit conversions: Use unit fractions: $\boxed{\pi = 180^\circ}$

$$90^\circ = 90^\circ\left(\frac{\pi}{180^\circ}\right)$$

$= \frac{\pi}{2}$

$\frac{\pi}{12} = \frac{\pi}{12}\left(\frac{180^\circ}{\pi}\right) = 15^\circ$

$225^\circ = 225^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{5\pi}{4}$

$-\frac{2\pi}{3} = -\frac{2\pi}{3}\left(\frac{180^\circ}{\pi}\right) = -120^\circ$

$17.6 = 17.6\left(\frac{180^\circ}{\pi}\right) \approx 1008^\circ$

$17.6^\circ = 17.6^\circ\left(\frac{\pi}{180^\circ}\right) \approx 0.307$

so, how big is exactly 1 rad in degrees?

$1 \text{ rad} = 1\left(\frac{180^\circ}{\pi}\right)$

$\approx 57.2957795\ldots^\circ$

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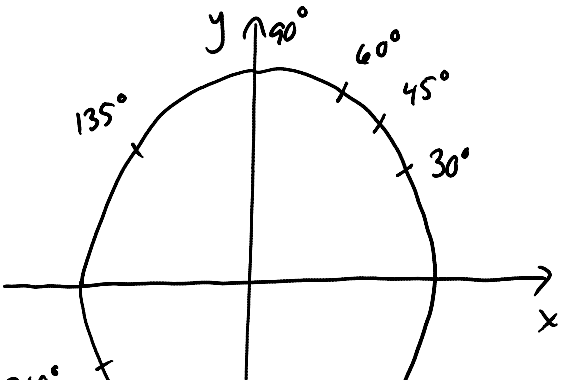
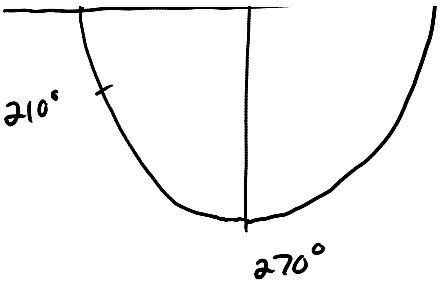
convert the following to rads:



$30^\circ = \pi/6$
$45^\circ = \pi/4$
$60^\circ = \pi/3$
$90^\circ = \pi/2$
$135^\circ = 3\pi/4$



$90° = \pi/2$

$135° = 3\pi/4$

$210° = 7\pi/6$

$270° = 3\pi/2$

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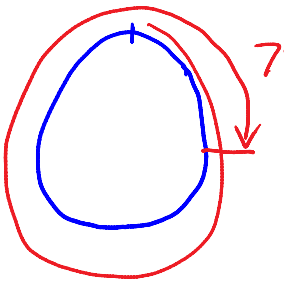
recall = $\theta = \frac{\text{arclength}}{\text{radius}} = \frac{s}{r}$

In 75 seconds, the tip of the classroom clock's second hand sweeps at a distance of 94 cm. What's the length of the second hand?



75 seconds corresponds to $\frac{5}{4}$ of a revolution

2π rads

$s = r\theta$

$r = \frac{s}{\theta} = \frac{94 \text{ cm}}{5\pi/2} = 11.9685 \text{ cm}$

$\approx 12 \text{ cm}$

The second hand is 12 cm long.