Section 7.1: Identities: Pythagorean and Sum/Difference

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identity ≡ an equation which is true for all possible values of the variable

examples:

$$2x = x + x$$

$$3(y+1) = 3y + 3$$

$$z^{-1} = \frac{1}{z}$$

note: although there is a place on the real number line where $1/z$ does not exist, for every z that does exist (the domain of $1/z$) this identity is true

Some trig identities we already know:

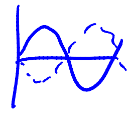
reciprocal trig identities

$$\csc x = \frac{1}{\sin x} \qquad \sin x = \frac{1}{\csc x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{1}{\tan x} \qquad \tan x = \frac{1}{\cot x}$$

also: negative identities

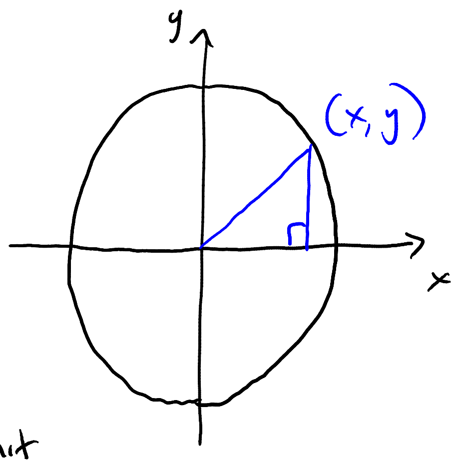


$$\sin(-x) = -\sin x \qquad \text{because } \sin x \text{ is } \underline{\underline{\text{odd}}}$$

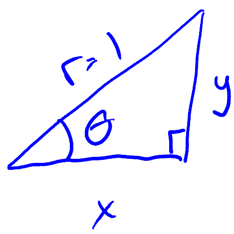
$$\cos(-x) = \cos x \qquad \cos x \text{ is } \underline{\text{even}}$$

$$\tan(-x) = -\tan x$$

Let's examine some new ones:



unit circle



$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

quotient identities:

$$\boxed{\begin{aligned} \tan\theta &= \frac{\sin\theta}{\cos\theta} \\ \cot\theta &= \frac{\cos\theta}{\sin\theta} \end{aligned}}$$

← this one is <u>extremely</u> useful!

Pythagorean identities:

continuing to look at the unit circle:

$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

note: $\cos^2\theta = (\cos\theta)^2$

$\cos^2\theta \neq \cos\theta^2$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

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there are two more:

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

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example: simplify

(1) $\sin^4\theta + \sin^2\theta\cos^2\theta$

$$\sin^2\theta\,(\underbrace{\sin^2\theta + \cos^2\theta}_{1})$$

$$\sin^2\theta$$