Section 7.1: cont'd

Wednesday, February 25, 2015
12:33 PM

trig substitution:

let $x = 2\sec\theta$ where θ is acute. substitute into $\sqrt{x^2-4}$ and simplify.

$$\sqrt{x^2-4} = \sqrt{(2\sec\theta)^2 - 4}$$
$$= \sqrt{4\sec^2\theta - 4}$$
$$= \sqrt{4(\sec^2\theta - 1)}$$
$$= \sqrt{4\tan^2\theta}$$
$$= 2\tan\theta$$

recall:
$\tan^2 x + 1 = \sec^2 x$
$\tan^2 x = \sec^2 x - 1$

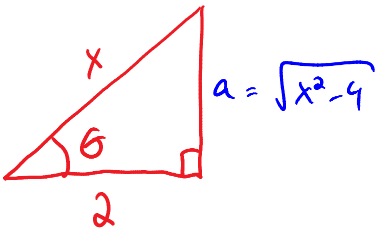
this example is like example 7 in textbook and questions 45-48, but using simpler language

note: if $x = 2\sec\theta$, what are $\sin\theta$, $\cos\theta$, and $\tan\theta$?

$$x = 2\sec\theta$$
$$\frac{x}{2} = \sec\theta$$

$$\cos\theta = \frac{1}{\sec\theta} = \boxed{\frac{2}{x}}$$

$\sec\theta$



$$a^2 + b^2 = c^2$$
$$a^2 + 4 = x^2$$
$$a^2 = x^2 - 4$$
$$a = \sqrt{x^2-4}$$

note: $\sqrt{x^2-4} \neq x - 2$

$$\boxed{\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{x^2-4}}{x}}$$

$$\boxed{\tan\theta = \frac{opp}{adj} = \frac{\sqrt{x^2-4}}{2}}$$

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sum & difference formulas: (derivation of this is shown in textbook)

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\boxed{\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B}$$

if using top sign here

then use top sign here also

we can use this quickly to prove stuff we already know:

example: use the sum/difference formulas to simplify

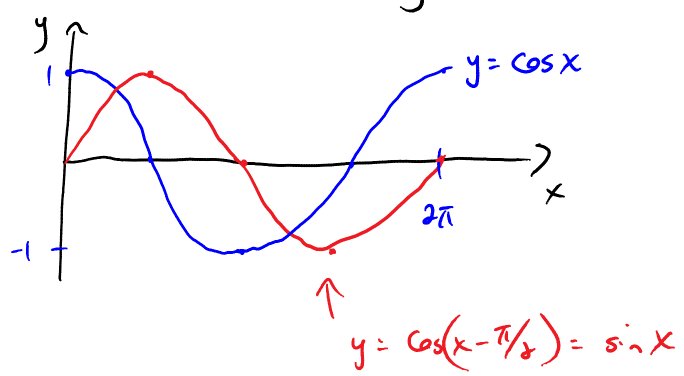
$$\cos\left(x - \pi/2\right)$$

$$\cos\left(x-\pi/2\right) = \cos x \, \cancelto{0}{\cos \pi/2} + \sin x \, \cancelto{1}{\sin \pi/2}$$

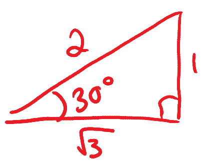
$$= 0 \qquad + \sin x$$

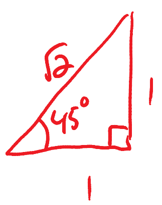
$$= \sin x$$

note: how did we already know this?



example: find $\cos 75°$ exactly





$\cos 45° = \frac{1}{\sqrt{2}}$

$$\cos 75° = \cos\left(30° + 45°\right)$$

$$= \cos 30° \cos 45° - \sin 30° \sin 45°$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2}\,\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

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also: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

also: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \qquad\qquad -$

and just look on your sheet for

$\tan(A \pm B)$

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example: simplify:

$$\frac{\sin(x-y) - \sin(x+y)}{\sin y}$$

$$\frac{(\sin x\cos y - \cos x \sin y) - (\sin x \cos y + \cos x \sin y)}{\sin y}$$

$$\frac{\cancel{\sin x\cos y} - \cos x \sin y - \cancel{\sin x \cos y} - \cos x \sin y}{\sin y}$$

$$-\frac{2\cos x \cancel{\sin y}}{\cancel{\sin y}}$$

$$-2\cos x$$

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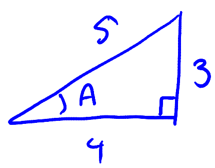
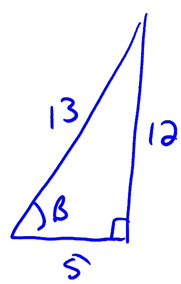
example: if $\sin A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$, where A and B are acute, calculate $\cos(A+B)$ exactly.





$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{4}{5}\,\frac{5}{13} - \frac{3}{5}\cdot\frac{12}{13}$$

$$= \frac{20}{65} - \frac{36}{65}$$

$$= \frac{-16}{65}$$

question: in what quadrant is the angle $(A+B)$?

it's in QII because $\cos(A+B)$ is negative