Section 7.2: Cofunction and Double-Angle Identities

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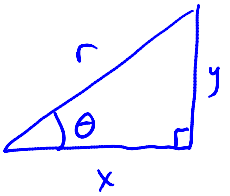
example: use the sum/difference identities to simplify:

$$\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$$

$$= \cos x$$

[note: this is our more familiar $\sin(90° - \theta) = \cos\theta$]



$$\cos\theta = \frac{x}{r}$$

$$\sin(90° - \theta) = \frac{x}{r}$$

there are six of these identities, called the cofunction identities, as listed on your sheet

same instructions:

$$\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2}$$

$$= -\cos x$$

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simplify: $\cot(x - 90°)$

$$\cot(x-90°) = \frac{\cos(x-90°)}{\sin(x-90°)}$$

$$= \frac{\sin x}{-\cos x}$$

$$\cos(x-90°) = \cos x \cos 90° + \sin x \sin 90°$$
$$= \sin x$$

$\sin(x-90°) = -\cos x$ because we just did that one!

$= \frac{\sin x}{-\cos x}$

$= -\tan x$

$\sin(x-90^\circ) = -\cos x$ because we just did that one!

Double-Angle Identities

how do we get them:

$$\sin 2\theta = \sin(\theta+\theta)$$
$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$
$$= \boxed{2\sin\theta\cos\theta}$$

$$\cos 2\theta = \cos(\theta+\theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$= \boxed{\cos^2\theta - \sin^2\theta}$$

$$= \cos^2\theta - (1-\cos^2\theta)$$
$$= \boxed{2\cos^2\theta - 1}$$

$$= \boxed{1 - 2\sin^2\theta}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$$

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example: if $\cos\theta = \frac{1}{3}$, calculate $\cos 2\theta$ <u>exactly</u>.

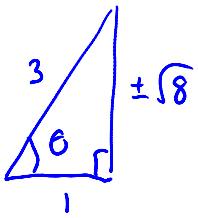
method #1: $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{1}{3}\right)^2 - 1$

$= 2/9 - 9/9$

$= -7/9$

method #2: $\cos 2\theta = \cos^2\theta - \sin^2\theta$



$= \left(\frac{1}{3}\right)^2 - \left(\frac{\pm\sqrt{8}}{3}\right)^2$

$= \frac{1}{9} - \frac{8}{9}$

$= \frac{-7}{9}$