Section 7.3: cont'd

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12:32 PM

prove:

$$\sec 2\theta \overset{?}{=} \frac{\sec^2\theta}{2-\sec^2\theta}$$

$$\frac{1}{\cos 2\theta} \quad \Bigg| \quad \frac{\frac{1}{\cos^2\theta}}{\left(\frac{\cos^2\theta}{\cos^2\theta}\right)2 - \frac{1}{\cos^2\theta}}$$

$$'' \quad \Bigg| \quad \frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta - 1}{\cos^2\theta}}$$

$$'' \quad \Bigg| \quad \frac{1}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{2\cos^2\theta - 1}$$

$$\frac{1}{2\cos^2\theta - 1} \quad \Bigg| \quad \frac{1}{2\cos^2\theta - 1}$$

✓ QED

note:

$$\frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}\left(\frac{\cos^2\theta}{\cos^2\theta}\right) = \frac{1}{2\cos^2\theta - 1}$$

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steps to proving trig identities:

① try swapping in another identity

$\sec 2\theta \rightarrow \frac{1}{\cos 2\theta}$

(2) try doing some algebra:

- factor
- combine into single fraction
- simplify
- expand (distribute)

... etc.

(3) (desperation step)

- change everything to sines & cosines

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prove:

$$\frac{\tan 3t - \tan t}{1 + \tan 3t \tan t} = \frac{2 \tan t}{1 - \tan^2 t}$$

$$\tan(3t - t) = \tan 2t$$

$$\tan 2t = \quad \checkmark$$

note: $\tan 3t - \tan t \neq \tan 2t$

$\sqrt{3t} - \sqrt{t} \neq \sqrt{2t}$

show that

$$\sec^4 m - \tan^2 m \overset{?}{=} \tan^4 m + \sec^2 m$$

method #1:

$$\sec^4 m - \tan^2 m = \tan^4 m + (\tan^2 m + 1)$$

$$(\tan^2 m + 1)^2 - \tan^2 m = \tan^4 m + \tan^2 m + 1$$

$$\tan^4 m + 2\tan^2 m + 1 - \tan^2 m =$$

$$\tan^4 m + \tan^2 m + 1 =$$

✓ QED

method #2:

$$\sec^4 m - \tan^2 m = \tan^4 m + \sec^2 m$$

$$\sec^4 m - (\sec^2 m - 1) = (\sec^2 m - 1)^2 + \sec^2 m$$

$$\sec^4 m - \sec^2 m + 1 = \sec^4 m - 2\sec^2 m + 1 + \sec^2 m$$

$$= \sec^4 m - \sec^2 m + 1$$

✓ QED

note: omit product-to-sum and sum-to-product identities

(also half-angle identities)