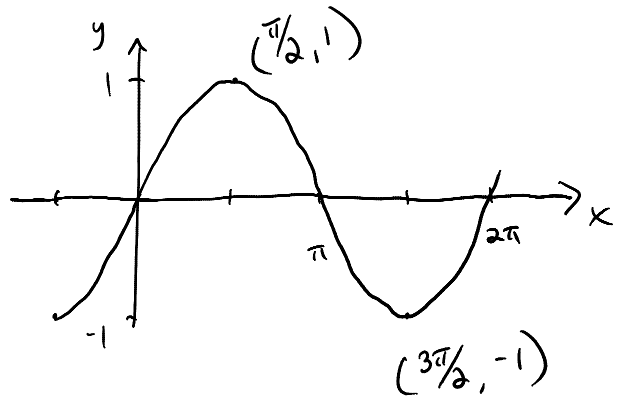
Section 7.4: Inverses of the Trig Functions

Friday, February 27, 2015
1:35 PM

sketch $y = \sin x$

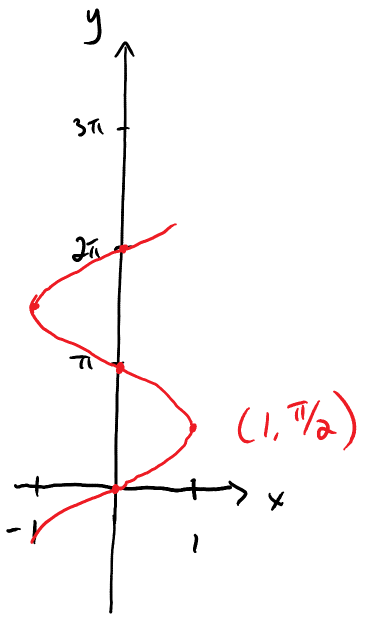


domain: $\mathbb{R}$

range: $[-1, 1]$

or $\{y \mid -1 \le y \le 1\}$

take inverse: (flip over $y = x$ line)



domain: $[-1, 1]$

range: $\mathbb{R}$

is it a function? no!

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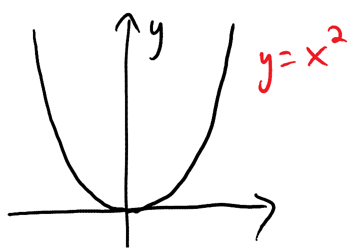
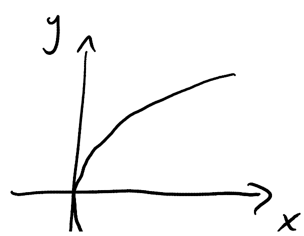
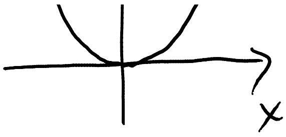
what if we really, really want/need the inverse to be a function?

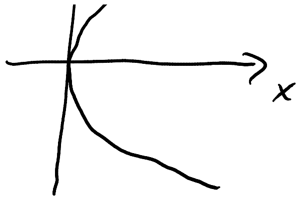


$\longleftrightarrow$





inverse

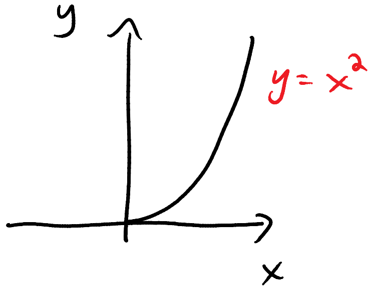


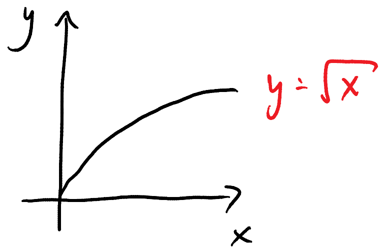
$x = y^2$

$y = \pm\sqrt{x}$

not a function

but

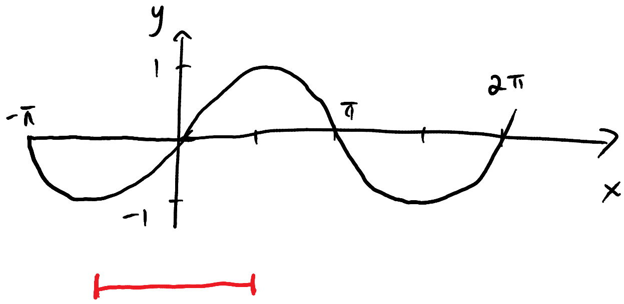




restrict domain to

$x \geq 0$

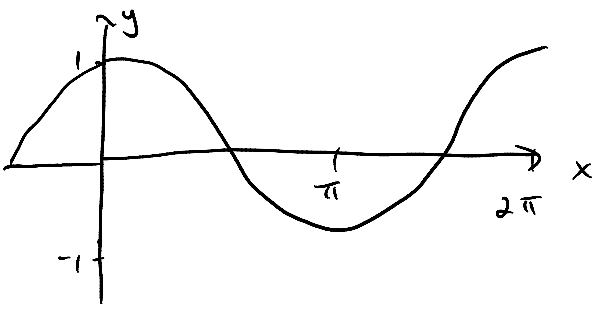
is a function



pick this domain restriction

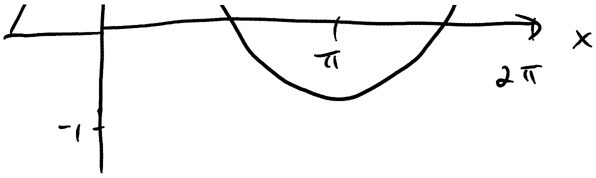
domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

range $[-1, 1]$



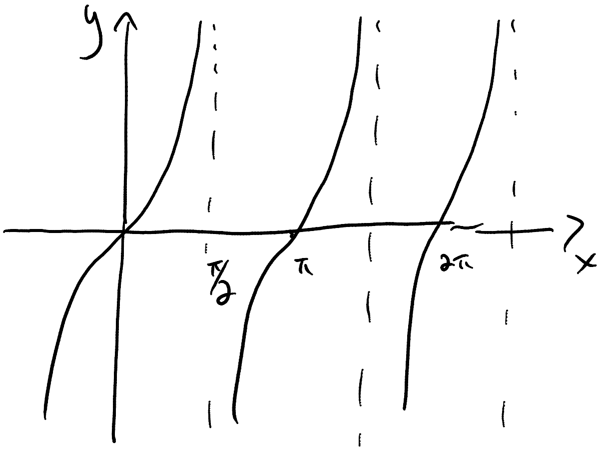
domain: $[0, \pi]$

range: $[-1, 1]$



domain : $[0, \ldots]$

range : $[-1, 1]$



domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

range : $\mathbb{R}$

examples: find $\tan^{-1}(-\sqrt{3})$

long, tedious method:

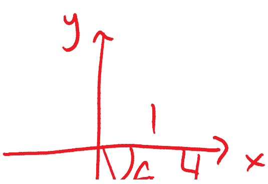
this is the same asking:

find θ if $\tan\theta = -\sqrt{3}$ and

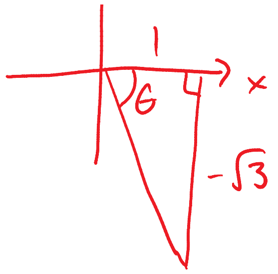
θ is in $(-90°, 90°)$

IV, I

so $\tan\theta$ is neg in II, IV



$\tan\theta = \frac{-\sqrt{3}}{1}$



this is a 30-60-90 triangle

so ref angle is $60°$

and $\theta = -60°$

short method: use calculator!

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find $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135°$

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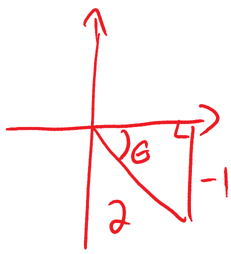
find $\sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

long, tedious method: find $\sin\theta$ where $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$



$\theta = -30°$

then $\sin\theta = \sin(-30°) = -\frac{1}{2}$

recall: $(f^{-1} \circ f)(x) = x$

$(f \circ f^{-1})(x) = x$

but have to be careful of domain restrictions!

restrictions!

example: $\cos^{-1}(\cos(-30°)) = \underbrace{30°}_{\text{from calculator}}$

why? next week!