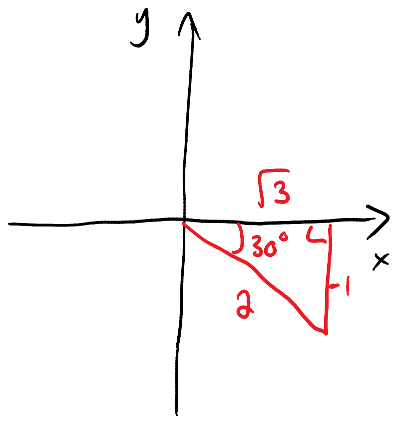
Section 7.4: cont'd

Monday, March 02, 2015
12:30 PM

$\cos^{-1}(\cos(-30°)) = 30°$

no negative here!



$\cos(-30°) = \frac{\sqrt{3}}{2}$

and $\cos^{-1}(\cos(-30°)) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

but recall: the range of $\cos^{-1}x$

is $[0, \pi]$ or $[0, 180°]$

but $\cos 30°$ is also $\frac{\sqrt{3}}{2}$

so $\cos^{-1}(\sqrt{3}/2) = 30°$

good news: your calculator will always get this right

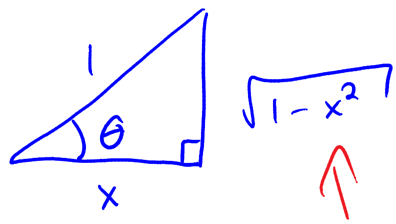
(appropriate use of technology!)

example: evaluate $\sin(\cos^{-1} x)$. Assume x is positive.

→ same as asking:

find $\sin\theta$ where $\theta = \cos^{-1} x$

so $\cos\theta = \frac{x}{1}$



Pythagorus

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$$

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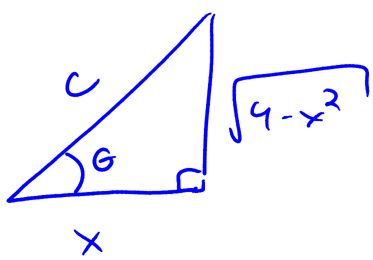
evaluate $\cos\left(\tan^{-1}\frac{\sqrt{4-x^2}}{x}\right)$ Assume x positive, & all angles in QI.

find $\cos\theta$ where $\theta = \tan^{-1}\frac{\sqrt{4-x^2}}{x}$

$\tan\theta = \frac{\sqrt{4-x^2}}{x}$



$$c^2 = a^2 + b^2$$
$$= x^2 + \left(\sqrt{4-x^2}\right)^2$$
$$= x^2 + 4 - x^2$$
$$= 4$$

$c = 2$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{2}$

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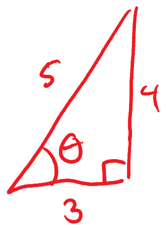
find $\sin\left(2\cos^{-1}\frac{3}{5}\right)$. Give an exact answer. (Assume all angles in QI.)

find $\sin 2\theta$ where $\theta = \cos^{-1}\left(\frac{3}{5}\right)$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5}$

$= \frac{24}{25}$

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find $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{3}{5}\right)$ exactly.

A $\quad$ B

$A = \sin^{-1}\left(\frac{1}{2}\right)$

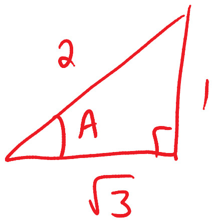
$\sin A = 1/2$

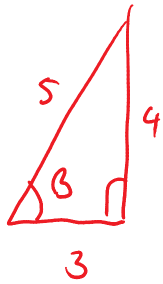
$B = \cos^{-1}\frac{3}{5}$

$\cos B = \frac{3}{}$

$\sin A = 1/2$

$\cos B = \frac{3}{5}$





$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{\sqrt{3}}{2} \cdot \frac{4}{5}$$

$$= \frac{3 + 4\sqrt{3}}{10}$$