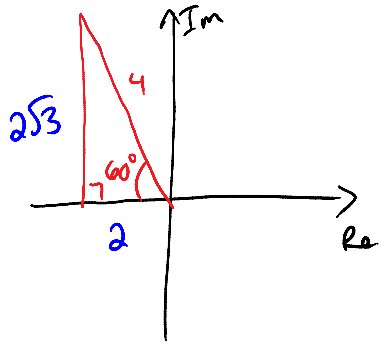
Section 8.3: cont'd

Friday, March 06, 2015
12:30 PM

Write the following complex numbers in the form $a+bi$:

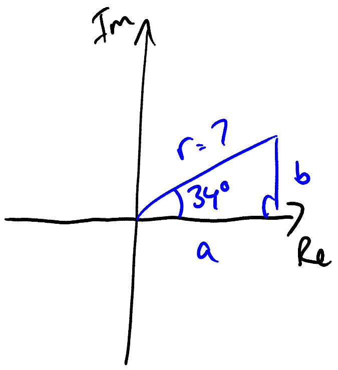
a) $r=4$, $\theta = 2\pi/3$ — give exact answers



$$a+bi = -2 + 2\sqrt{3}\,i$$
$$= -2 + 2i\sqrt{3}$$

} either

b) $r=7$, $\theta = 34°$ — round to one decimal place



$\cos 34° = \frac{a}{r}$

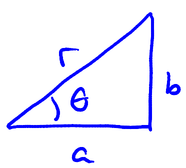
$a = r\cos 34°$
$= 7\cos 34°$
≈ 5.8032
≈ 5.8

$b = r\sin 34°$
$= 7\sin 34°$
≈ 3.914
≈ 3.9

$$a+bi = 5.8 + 3.9i$$

---

the general case:



if given a & b:

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a}$$

if given r and θ:

$$a = r\cos\theta$$

$$b = r\sin\theta$$

which means that

$$a + bi = r\cos\theta + r\sin\theta \cdot i$$

$$= \underbrace{r(\cos\theta + i\sin\theta)}_{\text{trig form of a complex number}}$$

so, using our previous results:

$$-5i = 5(\cos 270° + i\sin 270°)$$

$$-2 - 3i = 3.6(\cos 236.3° + i\sin 236.3°)$$

$$-2 + 2i\sqrt{3} = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$5.8 + 3.9i = 7(\cos 34° + i\sin 34°)$$

this notation is pretty annoying and cumbersome!

shorter, more compact notation:

$$\underbrace{a+bi}_{\text{rectangular form}} = \underbrace{r(\cos\theta + i\sin\theta)}_{\text{trig form}} = \underbrace{\begin{cases} re^{i\theta} \\ r\angle\theta \end{cases}}_{\text{phasor form}}$$

← angle form

---

why do we care?

let's calculate $(-2+2i\sqrt{3})^3$

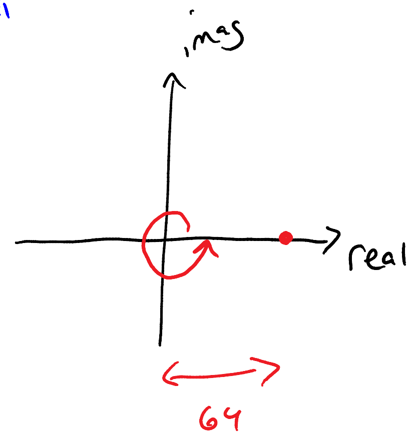
$$\begin{aligned}(-2+2i\sqrt{3})^3 &= \underbrace{(-2+2i\sqrt{3})(-2+2i\sqrt{3})}(-2+2i\sqrt{3}) \\ &= (4-4i\sqrt{3}-4i\sqrt{3}+4i^3 3)(-2+2i\sqrt{3}) \\ &= (4-8i\sqrt{3}-12)(-2+2i\sqrt{3}) \\ &= (-8-8i\sqrt{3})(-2+2i\sqrt{3}) \\ &= 16-\cancel{16i\sqrt{3}}+\cancel{16i\sqrt{3}}-16i^2 3 \\ &= 16+48 \\ &= 64\end{aligned}$$

but $(-2+2i\sqrt{3})^3 = (4e^{i2\pi/3})^3$

$$= 4^3 \left(e^{i 2\pi/3}\right)^3$$

$$= 64 \, e^{i 2\pi}$$

$$= 64$$



what about multiplication?

$$\left(2e^{i\pi/4}\right)\left(5e^{i\pi/6}\right)$$

$$= 10 \, e^{i\pi/4 + i\pi/6}$$

$$= 10 \, e^{i\pi(1/4 + 1/6)}$$

$$= 10 \, e^{i 5\pi/12}$$

division?

$$\frac{2e^{i\pi/4}}{5e^{i\pi/6}} = \frac{2}{5} e^{i(\pi/4 - \pi/6)}$$

$$= \frac{2}{5} e^{i\pi/12}$$

---

general cases:

$$(r_1 e^{i\theta_1})(r_2 e^{i\theta_2}) = r_1 r_2 e^{i(\theta_1+\theta_2)}$$

$$\frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$$

$$(r e^{i\theta})^n = r^n e^{in\theta}$$

note: $(r_1 \angle \theta_1)(r_2 \angle \theta_2) = r_1 r_2 \angle(\theta_1+\theta_2)$

---

do these shortcuts work for the trig form?

$$r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2 [\cos\theta_1 \cos\theta_2 + i\cos\theta_1 \sin\theta_2 + i\sin\theta_1\cos\theta_2 + i^2 \sin\theta_1 \sin\theta_2]$$

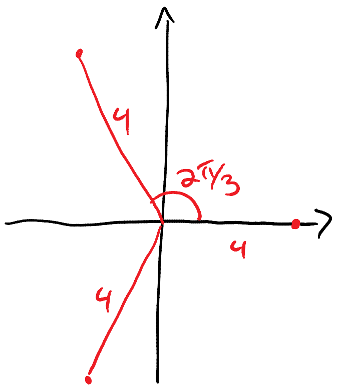
$$= r_1 r_2 [(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)]$$

$$= r_1 r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$$

yes!

---

digression:



$$(-2 + 2i\sqrt{3})^3 = 64$$