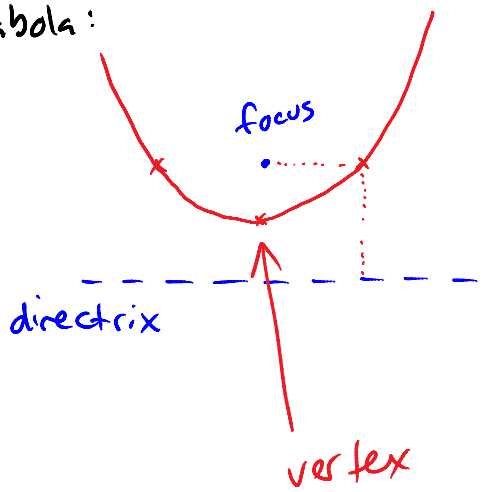
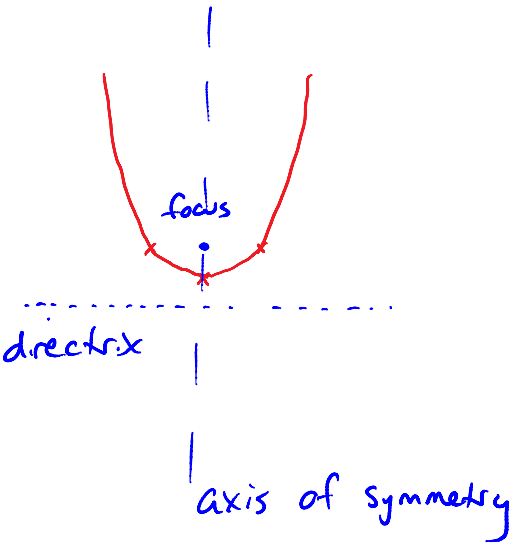
Section 10.1: cont'd

Monday, March 09, 2015
12:30 PM

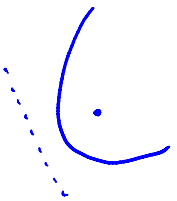
parabola:



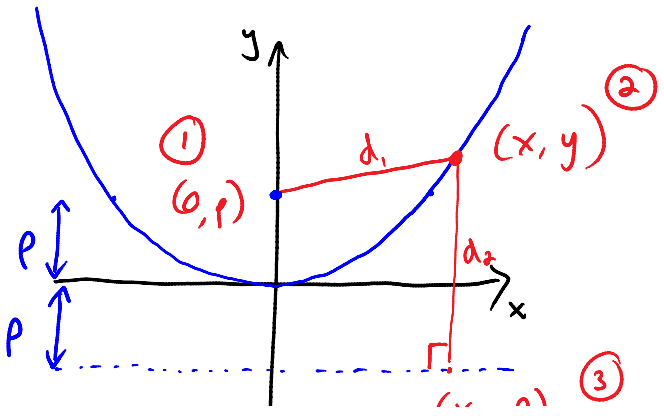


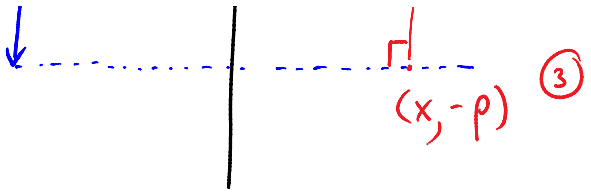
note: the "shallowness" of the parabola is determined by the distance between the focus and the directrix

note also: definition of parabola doesn't use algebra or the coordinate plane, so could also have



derivation of the equation of a parabola (digression):





$$d_1 = d_2$$

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x_2-x_3)^2+(y_2-y_3)^2}$$

$$\sqrt{(x-0)^2+(y-p)^2} = \sqrt{(x-x)^2+(y+p)^2}$$

$$\sqrt{x^2+y^2-2py+p^2} = \sqrt{0+y^2+2py+p^2}$$

$$x^2+y^2-2py+p^2 = y^2+2py+p^2$$

$$\boxed{x^2 = 4py}$$

note: you can rewrite this as

$$y = \frac{1}{4p}x^2$$

if you wish

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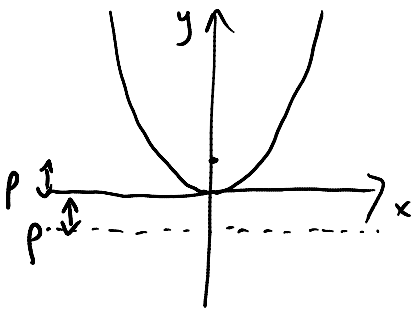
standard equation of parabola with vertex at origin:



$x^2 = 4py$ has vertex $(0,0)$

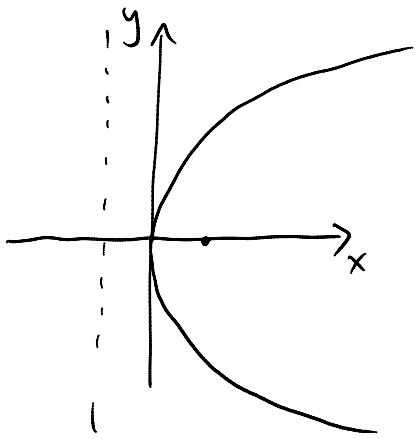
focus $(0,p)$

directrix $y = -p$

axis of symmetry: y-axis (or $x=0$)

note: if p is negative, then the focus is below the origin, the directrix is

above, and the parabola is directed downwards



$y^2 = 4px$ has vertex : $(0,0)$
focus : $(p,0)$
directrix : $x = -p$
axis of symmetry : x-axis $(y=0)$

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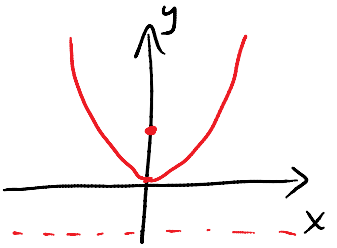
example: Locate the focus and directrix for each of the following parabolas:

a) $x^2 = 16y$

$x^2 = 4py$

$16 = 4p$

$p = 4$

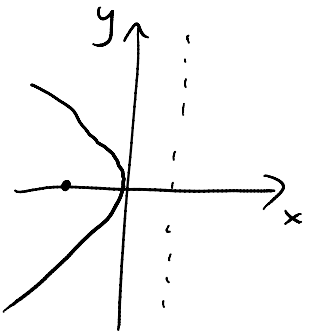


focus : $(0,4)$
directrix : $y = -4$

b) $2x + y^2 = 0$

$y^2 = -2x$

$y^2 = 4px$



$4p = -2$

$p = -\frac{1}{2}$

focus: $(-\frac{1}{2}, 0)$

directrix: $x = \frac{1}{2}$

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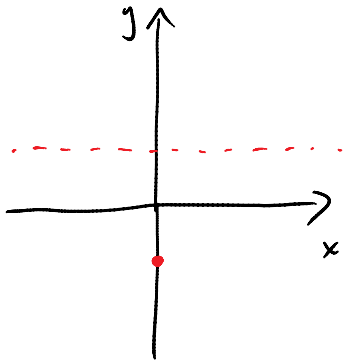
example: Give the equation of a parabola with focus at $(0, -\frac{1}{4})$ and directrix $y = \frac{1}{4}$.



So parabola opens downward with $p = -\frac{1}{4}$

then $x^2 = 4py$

$= 4(-\frac{1}{4})y$

$$x^2 = -y$$

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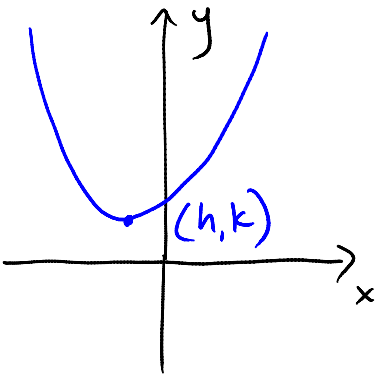
how can we make this more annoying?



$x^2 = 4py$

↓ shift

$$(x-h)^2 = 4p(y-k)$$

similarly,

$$(y-k)^2 = 4p(x-h)$$

note: $y = (x-1)^2 + 3$ (+3: shift up)

$y - 3 = (x-1)^2$ (shift up)

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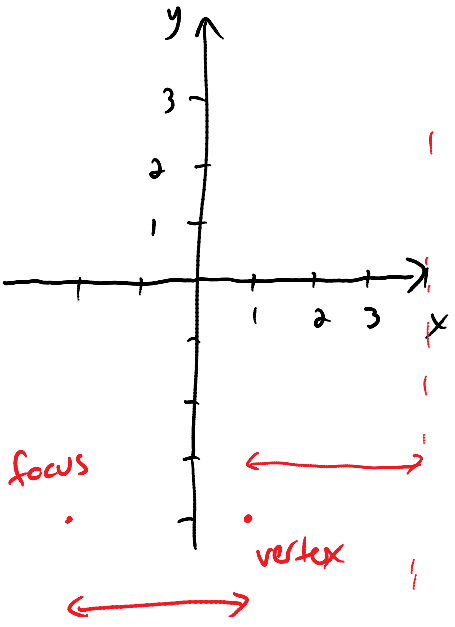
example: Find the vertex, focus, and directrix of the parabola

$$(y+4)^2 = -12(x-1)$$

$$(y-k)^2 = 4p(x-h)$$

so $h = 1$, $k = -4$, $p = -3$ } vertex is at $(1, -4)$

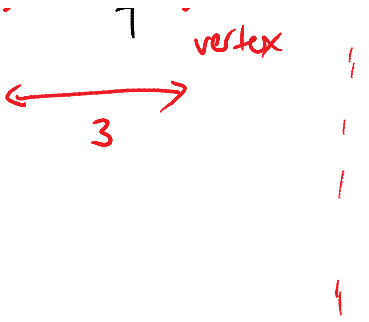


horizontal (y^2, not x^2)

opens to left (p is negative)

focus: $(-2, -4)$

directrix: $x = 4$



directrix : $\boxed{x = 4}$

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example: Find directrix, focus, and vertex of the parabola:

$$y^2 + 2y - 8x + 17 = 0$$

divide by 2 and square it →

$$y^2 + 2y + 1 = 8x - 17 + 1$$

$$(y+1)^2 = 8x - 16$$

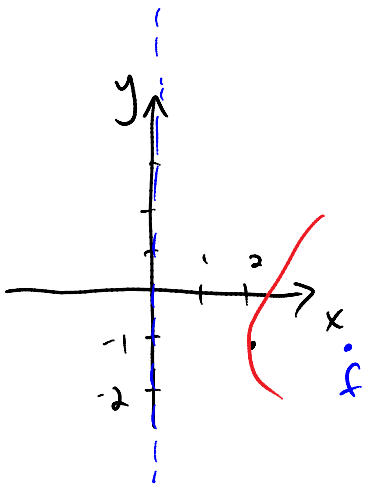
$$(y+1)^2 = 8(x-2)$$

$$(y-k)^2 = 4p(x-h)$$

so $h = 2$

$k = -1$

$p = 2$



vertex is $(2, -1)$

focus at $(4, -1)$

directrix $x = 0$ (y-axis)

Find directrix, focus, and vertex of parabola:

$$x^2 - x + y + 2 = 0$$

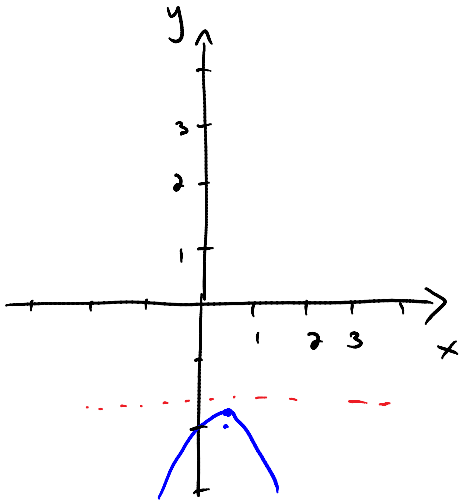
$$x^2 - x + \left(-\tfrac{1}{2}\right)^2 = -y - 2 + \left(-\tfrac{1}{2}\right)^2$$

$$\left(x - \tfrac{1}{2}\right)^2 = -y - 2 + \tfrac{1}{4}$$

$$\left(x - \tfrac{1}{2}\right)^2 = -y - \tfrac{7}{4}$$

$$\left(x - \tfrac{1}{2}\right)^2 = -1\left(y + \tfrac{7}{4}\right)$$

$$(x-h)^2 = 4p(y-k)$$



$h = \tfrac{1}{2}$

$k = -\tfrac{7}{4}$

$p = -\tfrac{1}{4}$

vertex: $\left(\tfrac{1}{2}, -\tfrac{7}{4}\right)$

focus: $\left(\tfrac{1}{2}, -2\right)$

directrix: $y = -\tfrac{3}{2}$