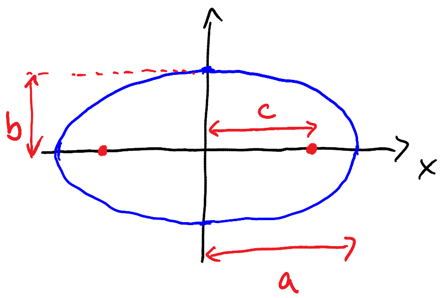
Section 10.2: cont'd

Wednesday, March 11, 2015
12:58 PM

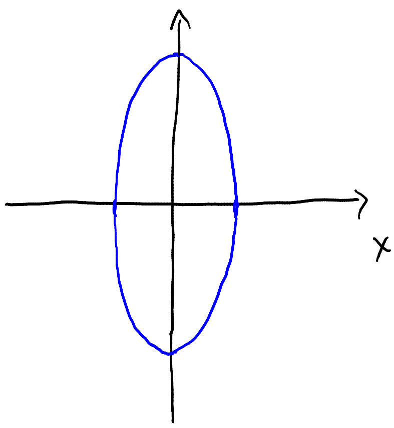
put on a coordinate system, centred at the origin:



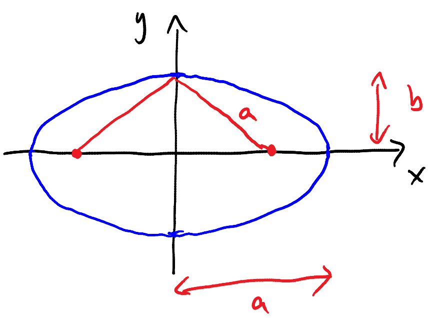
a = length of the semi-major axis
b = " " " semi-minor "

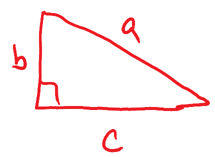
$a > b$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$



$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$





$c^2 + b^2 = a^2$

(sorry!)

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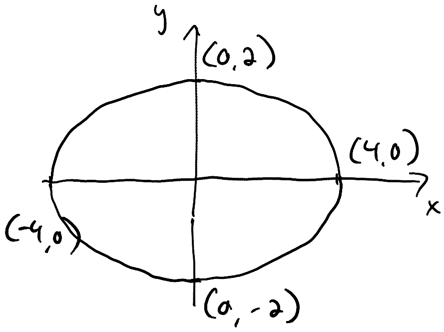
example: write the equation of the following ellipse and state the coordinates of the foci.



$a = 4$
$b = 2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\boxed{\frac{x^2}{16} + \frac{y^2}{4} = 1}$$

$$a^2 - b^2 = c^2$$
$$16 - 4 = c^2$$
$$c = \sqrt{12}$$
$$= 2\sqrt{3}$$

foci: $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$

or $(\pm 2\sqrt{3}, 0)$

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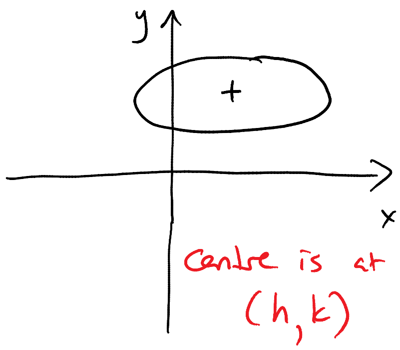
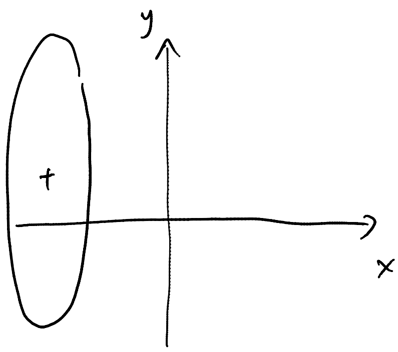
now translate:



centre is at (h, k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

how can you tell from the equation which orientation you have?

recall: $a > b$

and if a is under x, then it's horizontal

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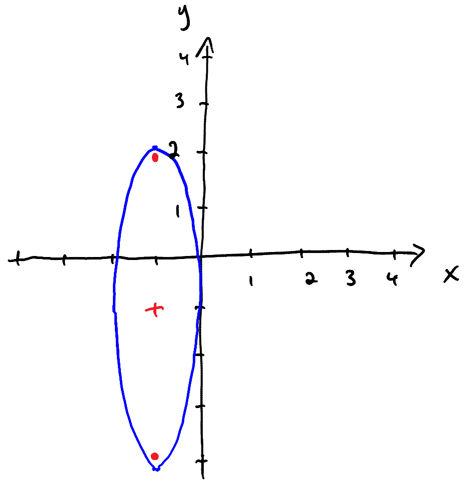
example: Find the equation of the ellipse with foci at $(-1, 2)$ and $(-1, -4)$ and with semi-minor axis of length 1. Also, state coordinates of centre and vertices. Lastly sketch it.



centre: $(-1, -1)$

$b = 1$

$c = 3$

$$a^2 = b^2 + c^2$$
$$= 1 + 9$$
$$= 10$$
$$a = \sqrt{10} \approx 3.2$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x+1)^2}{1} + \frac{(y+1)^2}{10} = 1$$

vertices: $(-1, -1 \pm \sqrt{10})$

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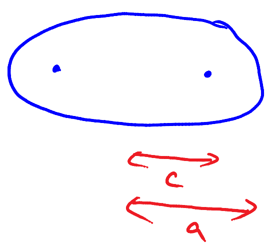
eccentricity: a measure of how circular an ellipse is



a $\quad\quad\quad\quad\quad\quad\quad\quad$ $\xleftrightarrow{a}$

$\varepsilon = \frac{c}{a}$ $\quad\quad\quad\quad$ $\varepsilon = \frac{c}{a}$

ε is close to 1 $\quad\quad\quad\quad$ ε is close to 0

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example: Calculate the eccentricity of the ellipse whose equation is:

$$64x^2 + 36y^2 - 128x + 360y - 1340 = 0$$

$$64x^2 - 128x + 36y^2 + 360y = 1340$$

$$64(x^2 - 2x + \underline{1}) + 36(y^2 + 10y + \underline{25}) = 1340 + \underline{64(1)} + \underline{36(25)}$$

$$\frac{64(x-1)^2}{2304} + \frac{36(y+5)^2}{2304} = \frac{2304}{2304}$$

$$\frac{(x-1)^2}{36} + \frac{(y+5)^2}{64} = 1$$

$a = 8$ $\leftarrow$ a is the bigger one!

$b = 6$

$$c^2 + b^2 = a^2$$
$$c^2 = a^2 - b^2$$
$$= 64 - 36$$
$$= 28$$
$$c = 2\sqrt{7}$$

$$\varepsilon = \frac{c}{a} = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4}$$