Section 11.1: cont'd

Monday, March 16, 2015
1:06 PM

recall: sequence - ordered list of numbers

example: 5, 8, 11, 14, ...

notation: terms are counted starting from 1:

$a_1, a_2, a_3, a_4, \ldots \quad a_n$

↑ index

three ways to define a sequence:

1. list all of the terms, or enough of the terms to set up the pattern

   note: if finite, need to give either the last term (or the total number of terms)

2. give a general formula

3. give a recursive formula

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general formula: a formula which allows you to compute the n^{th} term as a function of n only example: $a_n = 3n + 2$

what's the 100^{th} term? $n = 100$

$$a_{100} = 3(100) + 2 = 302$$

what's a_1? $a_1 = 3(1) + 2 = 5$

a_2? $a_2 = 8$

a_3? $a_3 = 11$

so the sequence $a_n = 3n + 2$ is the same sequence as $5, 8, 11, \ldots$

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recursive formula: a formula which gives a_n as a function of the previous term or terms

example: the sequence $5, 8, 11, \ldots$ has recursive formula

$$\begin{cases} a_1 = 5 & \leftarrow \text{need first term} \\ a_n = a_{n-1} + 3 & \leftarrow \text{need rule to get from } \underline{\text{any}} \text{ term to the next one} \end{cases}$$

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example: give the first three terms of

$$\begin{cases} a_1 = 3 \\ a_n = (a_{n-1})^3 - 10 \end{cases}$$

$a_1 = 3$

$a_2 = (a_1)^3 - 10 = 3^3 - 10 = 17$

$a_3 = (a_2)^3 - 10 = 17^3 - 10 = 4903$

3, 17, 4903

don't forget to include this one!

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most famous recursive formula (digression)

Fibonacci sequence: 1, 1, 2, 3, 5, 8, ...

recursive:

$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$$

general:

$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

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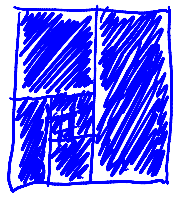
series: the sum of a sequence

examples: $16 + 20 + 24 + 28 + \dots 64$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$



notation: S_n = sum of the first n terms of a series

(if series is finite, could be sum of <u>all</u> terms)

S_∞ = sum of an infinite series

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example: for the series $16 + 20 + 24 + \dots 64$, calculate S_3 and S_5.

$S_3 = 16 + 20 + 24 = 60$

$S_5 = 16 + 20 + 24 + 28 + 32 = 120$

note: if n is large, this could get annoying! we'll develop more efficient methods later

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sigma notation: $\sum$ - Greek letter sigma (uppercase)

last value of index

Σ - "sum of"

5 1 2 3 4 5

$$\sum_{i=1}^{5} (3i+2) = \overset{1}{(3\cdot 1+2)} + \overset{2}{(3\cdot 2+2)} + \overset{3}{(3\cdot 3+2)} + \overset{4}{(3\cdot 4+2)} + \overset{5}{(3\cdot 5+2)}$$

first value of the index i

$$= 5 + 8 + 11 + 14 + 17$$

$$= 55$$

$$\sum_{j=8}^{10} (3\cdot 2^{10-j}) = \overset{8}{(3\cdot 2^{2})} + \overset{9}{(3\cdot 2^{1})} + \overset{10}{(3\cdot 2^{0})}$$

$$= 12 + 6 + 3$$

$$= 21$$

$$\sum_{k=20}^{24} 50 = \overset{20}{50} + \overset{21}{50} + \overset{22}{50} + \overset{23}{50} + \overset{24}{50}$$

$$= 250$$

how many terms? # terms = last − first + 1

indices