

# Section 11.2: Arithmetic Sequences and series

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11:44 AM

examples:

$$16, 21, 26, 31, 36, \dots, 141$$

$$95, 83, 71, 59, \dots$$

$$\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \dots, 10$$

pattern?

add 5

add -12

add  $\frac{1}{2}$

arithmetic sequence  $\equiv$  a sequence in which the next term is found by adding a constant to the previous term

$\rightarrow$  common difference  $d$

example: write a recursive formula for the sequence  
 $-7, -2, 3, 8, 13, \dots$

$$\begin{cases} a_1 = -7 \\ a_n = a_{n-1} + 5 \end{cases} \quad \leftarrow \text{or } a_{n+1} = a_n + 5$$

example: write a general formula for the sequence  
 $2, 5, 8, 11, \dots$

①

②

③

④

⑤

^

①	②	③	④	...	①
2,	5,	8,	11,	...	$a_n$
2,	$2+3,$	$2+3 \cdot 2,$	$2+3 \cdot 3,$	...	$2+3 \cdot (n-1)$

for 2, 5, 8, 11, ...

$$a_n = 2 + 3(n-1)$$

$$= 2 + 3n - 3$$

$$= 3n - 1$$

← simplified version

in general,

$$a_n = a_1 + (n-1)d$$

example: how many terms does the following sequence have?

$$4, 7, 10, 13, \dots, 301$$

arithmetic with  $d=3$

$$a_n = a_1 + (n-1)d$$

$$301 = 4 + (n-1)3$$

$$297 = 3(n-1)$$

$$99 = n-1$$

$$n = 100$$

example: Find the first four terms of the arithmetic sequence in which the twentieth term is 139 and the thirty-fifth term is 229.

$$a_n = a_1 + (n-1)d$$

$$a_{20} = 139$$

$$139 = a_1 + (20-1)d$$

$$139 = a_1 + 19d$$

mult  
by -1

$$-139 = -a_1 - 19d$$

$$229 = a_1 + 34d$$

$$90 = 15d$$

$$d = 6$$

$$139 = a_1 + 19d$$

$$139 = a_1 + 19(6)$$

$$a_1 = 25$$

$$a_{35} = 229$$

$$229 = a_1 + (35-1)d$$

$$229 = a_1 + 34d$$

25, 31, 37, 43

arithmetic series:

$$2 + 5 + 8 + 11 + \dots$$

example: calculate, for the above series,  $S_6$  and  $S_7$

$$\begin{aligned} S_6 &= 2 + 5 + 8 + 11 + 14 + 17 \\ &= 57 \end{aligned}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_7 = 2 + 5 + 8 + 11 + 14 + 17 + 20$$

Diagram illustrating the sum of an arithmetic series with 7 terms. Brackets indicate that the sum of the first and last terms (2 + 20) is 22, the sum of the second and second-to-last terms (5 + 17) is 22, and the sum of the third and third-to-last terms (8 + 14) is 22. A larger bracket under the entire series also indicates a total sum of 22.

note:  $S_n$  formula works for both even and odd  $n$

arithmetic series:

$$S_n = \frac{n}{2} (a_1 + a_n)$$
$$= \frac{n}{2} [2a_1 + (n-1)d]$$