Section 11.2: Arithmetic Sequences and Series

Tuesday, March 17, 2015
11:44 AM

examples:

pattern?

16, 21, 26, 31, 36, ... 141 — add 5

95, 83, 71, 59, ... — add −12

$\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \ldots 10$ — add ½

arithmetic sequence ≡ a sequence in which the next term is found by adding a constant to the previous term

common difference d

example: write a recursive formula for the sequence
−7, −2, 3, 8, 13, ...

$$\begin{cases} a_1 = -7 \\ a_n = a_{n-1} + 5 \end{cases}$$

← or $a_{n+1} = a_n + 5$

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example: write a general formula for the sequence
2, 5, 8, 11, ...

1 2 3 4 n

$\textcircled{1}$ $\quad$ $\textcircled{2}$ $\quad$ $\textcircled{3}$ $\quad$ $\textcircled{4}$ $\quad$ $\textcircled{n}$

$$2, \quad 5, \quad 8, \quad 11, \quad \ldots \quad a_n$$

$$2, \quad 2+3, \quad 2+3\cdot 2, \quad 2+3\cdot 3, \quad \ldots \quad 2+3\cdot(n-1)$$

for $2, 5, 8, 11, \ldots$

$$a_n = 2 + 3(n-1)$$
$$= 2 + 3n - 3$$
$$= 3n - 1$$

← simplified version

in general, $\boxed{a_n = a_1 + (n-1)d}$

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example: how many terms does the following sequence have?

$$4, 7, 10, 13, \ldots 301$$

arithmetic with $d = 3$

$$a_n = a_1 + (n-1)d$$
$$301 = 4 + (n-1)3$$
$$297 = 3(n-1)$$
$$99 = n-1$$
$$n = 100$$

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example: Find the first four terms of the arithmetic sequence in which the twentieth term is 139 and the thirty-fifth term is 229.

$$a_n = a_1 + (n-1)d$$

$a_{20} = 139$

$a_{35} = 229$

$139 = a_1 + (20-1)d$

$139 = a_1 + 19d$

$229 = a_1 + (35-1)d$

$229 = a_1 + 34d$

mult by -1

$$\begin{aligned} -139 &= -a_1 - 19d \\ 229 &= a_1 + 34d \\ \hline 90 &= 15d \end{aligned}$$

$d = 6$

$139 = a_1 + 19d$

$139 = a_1 + 19(6)$

$a_1 = 25$

$\boxed{25, 31, 37, 43}$

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arithmetic series:

$2 + 5 + 8 + 11 + \ldots$

example: calculate, for the above series, S_6 and S_7

$S_6 = 2 + 5 + 8 + 11 + 14 + 17$

19

19

19

$= 57$

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

$$S_7 = \underbrace{2 + \overbrace{5 + \underbrace{8 + 11 + 14}_{22} + 17}^{22} + 20}_{22}$$

note: S_n formula works for both even and odd n

arithmetic series:

$$\boxed{\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_n) \\ &= \frac{n}{2}\left[2a_1 + (n-1)d\right] \end{aligned}}$$