Section 11.3: Geometric sequences and series

Wednesday, March 18, 2015
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examples:

		pattern?
1	$7, 14, 28, 56, \ldots$	mult by 2
2	$100, 20, 4, 4/5, \ldots$	mult by 1/5
3	$1/2, 1/4, 1/8, 1/16, \ldots\ 1/1024$	mult by 1/2
4	$24, -16, 32/3, -64/9, \ldots$	mult by $-2/3$
5	$-1, 1, -1, 1, -1, 1, \ldots$	mult by -1

geometric sequence $\equiv$ a sequence in which the next term is just the previous term multiplied by a constant

↳ common ratio r

how to find r? take any term and divide by previous one

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$$r = \frac{-16}{24} = \frac{-2}{3}$$

$$= \frac{32/3}{-16} = \frac{32}{3} \cdot \frac{1}{-16} = \frac{-2}{3}$$

or ...

example: find a recursive formula for the sequence $100, 20, 4, 4/5, \ldots$

$$\begin{cases} a_1 = 100 \\ a_n = \frac{1}{5} a_{n-1} \end{cases} \qquad \left(\text{or } \frac{a_{n-1}}{5} \text{ or } 0.2\, a_{n-1} \right)$$

example: find a general formula for the same sequence

1	2	3	4		n
100,	20,	4,	$4/5$,	...	a_n
100,	$100\left(\frac{1}{5}\right)$,	$100\left(\frac{1}{5}\right)^2$,	$100\left(\frac{1}{5}\right)^3$,	...	$100\left(\frac{1}{5}\right)^{n-1}$

for $100, 20, 4, 4/5, \ldots$ $\quad a_n = 100\left(\frac{1}{5}\right)^{n-1}$

in general, $\boxed{a_n = a_1 r^{n-1}}$ for geometric

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example: find the 12^{th} term in the sequence
$5, 15, 45, 135, \ldots$

geometric with $r = 3$

$$\begin{aligned} a_n &= a_1 r^{n-1} \\ &= 5 \cdot 3^{11} \\ &= 885735 \end{aligned}$$

(note: the 50^{th} term is 1.19×10^{24})

geometric series:

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

digression: why?

$$S_n = a_1 + a_2 + a_3 + \dots \quad a_{n-1} + a_n$$

add

$$S_n = a_1 + a_1 r + a_1 r^2 + \dots \quad a_1 r^{n-2} + a_1 r^{n-1}$$

$$-rS_n = -a_1 r - a_1 r^2 - a_1 r^3 - \dots - a_1 r^{n-1} - a_1 r^n$$

$$S_n - rS_n = a_1 - a_1 r^n$$

$$S_n(1-r) = a_1(1-r^n)$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

evaluate the sum of the first 50 terms of

$$2 + 10 + 50 + \dots$$

geometric $r = 5$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$= \frac{2(1-5^{50})}{1-5}$$

$$= 4.4 \times 10^{34}$$

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but what about $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$?

if we naively substitute:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$r = 1/4$

r^∞ ? $\quad \left(\frac{1}{4}\right)^\infty$?

as $n \to \infty$, $\left(\frac{1}{4}\right)^n \to 0$

so $\boxed{S_\infty = \frac{a_1}{1-r} \quad \text{provided that } |r| < 1}$

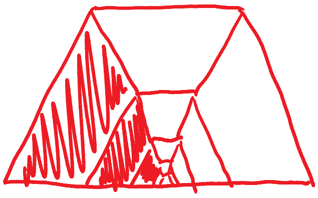
back to $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

geometric $\quad r = 1/4 \qquad |r| < 1$? yes!

$a_1 = 1/4$

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$$



evaluate $\sum_{j=0}^{\infty} 75\left(\frac{3}{5}\right)^j = 75 + 75\left(\frac{3}{5}\right) + 75\left(\frac{3}{5}\right)^2 + \dots$

0 1 2

geometric with $r = 3/5$, $a_1 = 75$ $|r| < 1$? ✓

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{75}{1-3/5} = 75 \cdot \frac{5}{2} = 187.5 \text{ or } \frac{375}{2}$$

evaluate: $12 + 18 + 27 + \dots$

geometric with $r = \frac{3}{2}$ but $|r| < 1$? No!

$$\cancel{S_\infty = \frac{a_1}{1-r} = \frac{12}{1-3/2} = \frac{12}{-1/2}} = -24$$

S_∞ = undefined

evaluate: $0.\overline{4} = 0.44444\dots$

$= 0.4 + 0.04 + 0.004 + 0.0004 + \ldots$

geometric with $r = \frac{1}{10}$ $\quad |r| < 1$? ✓

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{4/10}{1 - 1/10} = \frac{4/10}{9/10} = \frac{4}{10} \cdot \frac{10}{9} = \frac{4}{9}$$