

Math 173 – Quiz #4

March 11, 2016

Name: Solution Set

Instructor: Patricia Wrean

Total: 40 points

1. Convert the angles in radians to degrees and the angles in degrees to radians. Show your work. If your answer is in radians, leave it as a multiple of π . (4 points)

a) $-\frac{13\pi}{12} = \frac{-13\pi}{12} \cdot \left(\frac{180^\circ}{\pi}\right) = \underline{-195^\circ}$

b) $300^\circ = 300^\circ \left(\frac{\pi}{180^\circ}\right) = \underline{\frac{5\pi}{3}}$

2. Use a calculator to evaluate the following. Round to two decimal places if the answer is approximate. (3 points)

a) $\cos\left(-\frac{\pi}{8}\right) \approx 0.92388 = \underline{0.92}$

b) $\tan 45 \approx 1.61978 = \underline{1.62}$

c) $\csc 1 = \frac{1}{\sin 1} \approx 1.1884 = \underline{1.19}$

3. Use the sum and/or difference identities to find an equivalent expression for $\sec\left(\frac{\pi}{2} - x\right)$. (4 points)

$$\begin{aligned}\cos\left(\frac{\pi}{2} - x\right) &= \overset{\textcircled{1}}{\cos\frac{\pi}{2}} \overset{\textcircled{1}}{\cos x} + \overset{\textcircled{1}}{\sin\frac{\pi}{2}} \overset{\textcircled{1}}{\sin x} \\ &= \sin x \quad \textcircled{1}\end{aligned}$$

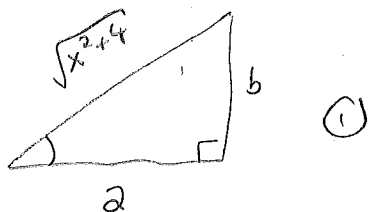
so $\sec\left(\frac{\pi}{2} - x\right) = \frac{1}{\cos\left(\frac{\pi}{2} - x\right)} = \boxed{\frac{1}{\sin x} \text{ or } \csc x}$ $\textcircled{1}$

4. Evaluate. Assume that x is positive and any angles would be in the first quadrant. (4 points)

$$\sin\left(\cos^{-1}\left(\frac{2}{\sqrt{x^2+4}}\right)\right)$$

find $\sin \theta$ where $\theta = \cos^{-1} \frac{2}{\sqrt{x^2+4}}$ ①

$$\cos \theta = \frac{2}{\sqrt{x^2+4}}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 4 + b^2 &= (\sqrt{x^2+4})^2 \\ 4 + b^2 &= x^2 + 4 \\ b^2 &= x^2 \rightarrow b = x \end{aligned} \quad \text{①}$$

$$\therefore \sin \theta = \frac{x}{\sqrt{x^2+4}} \quad \text{①}$$

① (ok)
 $\sin x = \frac{x}{\sqrt{x^2+4}}$ ①
 $\sin \frac{x}{\sqrt{x^2+4}}$ ①
 $\sin = \frac{x}{\sqrt{x^2+4}}$ wrong

5. Simplify.

(5 points)

$$\frac{2 \tan \theta}{1 - \tan^2 \theta} - \frac{2 \sin \theta \cos \theta}{1 - 2 \cos^2 \theta}$$

$$\underline{2 \tan 2\theta}$$

$$\textcircled{1} \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} \quad \textcircled{1}$$

$$= \frac{\sin 2\theta}{2 \cos^2 \theta - 1} \quad \textcircled{1}$$

$$\tan 2\theta + \frac{\sin 2\theta}{\cos 2\theta} \quad \textcircled{1}$$

$$\tan 2\theta + \tan 2\theta$$

$$2 \tan 2\theta \quad \textcircled{1}$$

6. Prove the following trig identity.

(5 points)

$$\frac{\cos x}{\csc x - \sin x} = \frac{1}{\cot x}$$

$$\textcircled{1} \left(\frac{\sin x}{\sin x} \right) \frac{\cos x}{\frac{1}{\sin x} - \sin x} = \tan x \quad \textcircled{1}$$

$$\textcircled{1} \frac{\sin x \cos x}{1 - \sin^2 x} = "$$

$$\textcircled{1} \frac{\sin x \cos x}{\cos^2 x} = "$$

$$\frac{\sin x \cancel{\cos x}}{\cos^{\cancel{2}} x} = "$$

$$\textcircled{1} \left\{ \begin{array}{l} \frac{\sin x}{\cos x} = \\ \tan x = \end{array} \right. =$$

7. Solve, finding all solutions in $[0, 2\pi)$ or $[0, 360^\circ)$. Give exact answers.

(5 points)

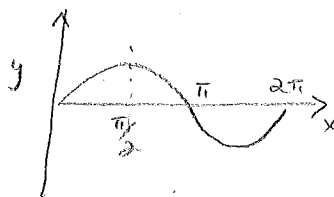
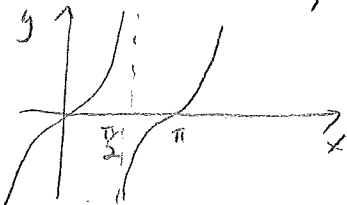
$$\sin x \tan x = \tan x$$

$$\underline{\{0, 180^\circ\}} \text{ or } \underline{\{0, \pi\}}$$

$$\sin x \tan x - \tan x = 0$$

$$\textcircled{2} \tan x (\sin x - 1) = 0$$

$$\textcircled{1} \begin{array}{l} \tan x = 0 \quad \leftarrow \quad \rightarrow \\ x = 0, 180^\circ \end{array} \quad \text{or} \quad \begin{array}{l} \sin x = 1 \\ x = 90^\circ \quad \textcircled{1} \end{array}$$



0 and 360° $\textcircled{-\frac{1}{2}}$

360° but not 0 $\textcircled{-\frac{1}{2}}$

270° $\textcircled{-1}$

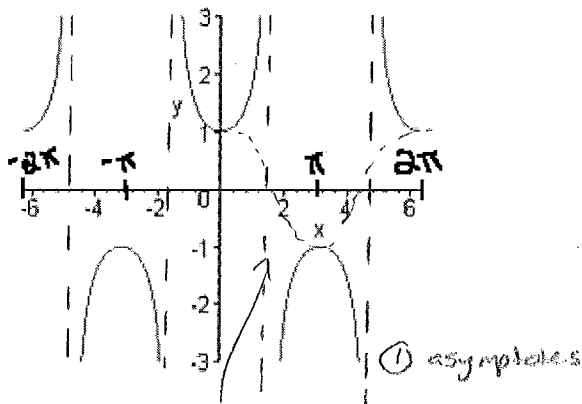
90° & 270° $\textcircled{-2}$

$$x = 0, \cancel{90^\circ}, 180^\circ$$

↑
extraneous because $\tan 90^\circ$ is undefined

$\textcircled{1}$

8. Consider the graph below. State which of the six basic trig functions it is by giving the equation of the graph. Calculate the function's period and range and sketch in the positions of any asymptotes. Is this function even, odd, or neither? (5 points)



$y = \cos x$

equation: $f(x) = \sec x$ (1)

period: 2π (1)

range: $(-\infty, -1] \cup [1, \infty)$ (1)

(1) even (symmetric w.r.t y-axis)

9. Sketch a graph of the function $y = 2 \cos\left(\frac{x}{3}\right)$, and state the function's period and amplitude. Include at least one full period in your sketch. State whether the function is even, odd, or neither. (5 points)

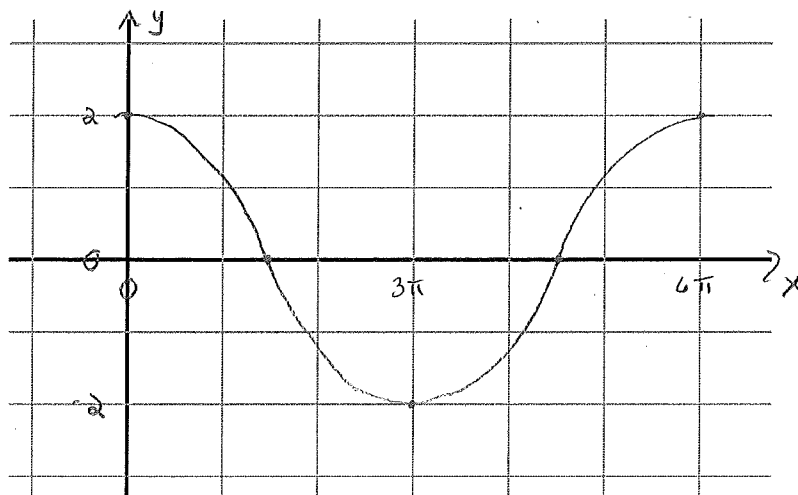
period = $\frac{2\pi}{|B|} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot \frac{3}{1} = 6\pi$

amplitude = $|A| = 2$

period: 6π (1)

amplitude: 2 (1)

even (1)



(-1) sine graph