Math 173 – Quiz #2

February 5, 2015 **Name:** Solution Set
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Total: 40 points

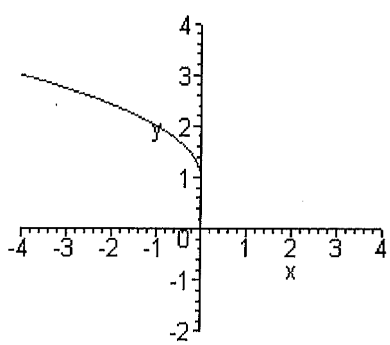
1. For $f(x) = x^3 - 4$ and $g(x) = \sqrt[3]{4-x}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. (4 points)

$(f \circ g)(x) = \left(\sqrt[3]{4-x}\right)^3 - 4$
$= 4 - x - 4$
$= -x$

$(g \circ f)(x) = \sqrt[3]{4-(x^3-4)}$
$= \sqrt[3]{8-x^3}$

mix up fog & gof: -1

2. Write the equation for the function given in the graph below. (Just in case it's not clear, the graph starts at (0,1) and then moves up and off to the left.) (3 points)



basic graph is $y = \sqrt{x}$ (1)
flipped over y is $y = \sqrt{-x}$ (1)
and shifted up is

$\boxed{y = \sqrt{-x} + 1}$ (1)

3. Consider the following polynomial: (3 points)

$f(x) = -3(x+1)^2(x-2)^3$

a) Find the zeros of this polynomial and their multiplicities.

mult 2

$x = -1, 2$ ← mult 3

(2)

b) Find the y-intercept.

$y = f(0) = -3(1)^2(-2)^3$
$= -3(-8)$
$= 24$

$\boxed{(0, 24)}$ (1)

4. Rewrite the equation of the following parabola in the form $f(x) = a(x-h)^2 + k$ **by completing the square** and state the equation of the axis of symmetry and the coordinates of the vertex. Is the vertex at the maximum or minimum point in the parabola (circle one)? (6 points)

$$f(x) = \frac{1}{2}x^2 - 4x + 11$$

$$= \tfrac{1}{2}(x^2 - 8x + \underline{16}) + 11 - \underline{\tfrac{1}{2}(16)}$$

$$= \tfrac{1}{2}(x-4)^2 + 3$$

2

equation: $f(x) = \tfrac{1}{2}(x-4)^2 + 3$ 1

vertex: $(4, 3)$ 1

axis of symmetry: $x = 4$ 1

maximum / minimum 1

5. Factor the following polynomial into linear factors. (5 points)

$$f(x) = x^3 - 5x + 2$$

$\frac{p}{q} = \pm 1, \pm 2$ 1

$f(1) \neq 0$

$f(-1) = -1 + 5 + 2 \neq 0$

$f(2) = 8 - 10 + 2 = 0$

so $(x-2)$ is a factor 1

$$\begin{array}{r} x^2 + 2x - 1 \\ x - 2 \overline{)\, x^3 + 0x^2 - 5x + 2} \\ \underline{x^3 - 2x^2} \\ 2x^2 - 5x \\ \underline{2x^2 - 4x} \\ -x + 2 \end{array}$$ 1

but $x^2 + 2x - 1$ doesn't factor (into rationals)

so set to zero and

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 + 4}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= -1 \pm \sqrt{2}$$ 1

$$f(x) = (x-2)(x+1+\sqrt{2})(x+1-\sqrt{2})$$

1

6. Consider the following rational function: (10 points)

$$f(x)=\frac{x^2+5x+4}{x+2}. = \frac{(x+1)(x+4)}{(x+2)}$$

a) What is the y-intercept?

set $x=0$: $y = f(0) = 4/2 = 2$

$\boxed{(0,2)}$ 1

b) What are the x-intercept(s)?

set num $=0$ $x=-1,-4$

$\boxed{(-1,0) \text{ and } (-4,0)}$ 2

c) Are there any vertical asymptotes? If so, where?

set denom $=0$

$\boxed{\text{yes, at } x=-2}$ 1

d) Are there any horizontal asymptotes? If so, where?

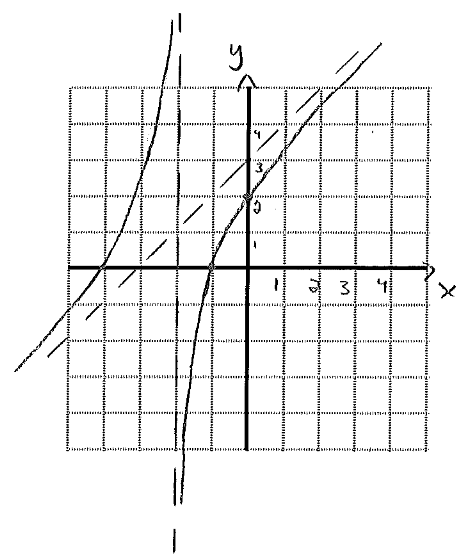
no 1

e) Are there any oblique/slant asymptotes? If so, where?

$$\begin{array}{r} x+3 \\ x+2\,\overline{)\,x^2+5x+4} \\ x^2+2x \\ \hline 3x+4 \end{array}$$

$\boxed{\text{yes, at } y=x+3}$ 1

f) Sketch the graph as accurately as possible.



4

7. Is the function $f(x)$ even, odd, or neither? Show your work. (2 points)

$$f(x)=\frac{x-1}{x^2}$$

neither

$$f(-x) = \frac{-x-1}{x^2} \neq -f(x)$$
$$\neq f(x)$$

8. Using Descartes' Rule, how many positive real zeros and negative real zeros can the following polynomial have? <u>Do not solve it!</u> (2 points)

$$f(x)=3x^7-2x^4+4x-2$$

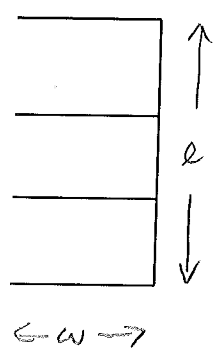
positive real zeros: 3 or 1

negative real zeros: 0

$$f(-x) = -3x^7-2x^4-4x-2$$

no sign changes

9. A rancher needs to enclose a rectangular field next to a river. He also wants to divide the field into three sections by adding two additional pieces of fence, as shown in the diagram. What is the maximum area that the rancher can enclose with ~~36~~ 56 metres of fence? (5 points)



$P = l + 4w$ ½

$36 = l + 4w$

$l = 56 - 4w$ 1

$A = lw$ ½

$= (36-4w)w$

$= 56w - 4w^2$ 1

method #1:

$A = 36w - 4w^2$

$w_{max} = \frac{-b}{2a} = \frac{-56}{-8}$

$= 7$

$l_{max} = 56 - 28 = 28$

$A_{max} = lw = \boxed{196 m^2}$

method #2:

$A = -4w^2 + 56w$

$= -4(w^2 - 14w + \underline{49}) - (-4)(49)$

$= -4(w-7)^2 + 196$

$\boxed{A_{max} = 196 m^2}$

2