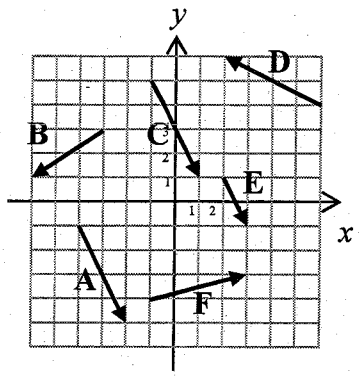
Math 185 – Assignment #1 **Name:** Solution Set

Total: 40

1. Consider vector **A** in the diagram below. List all of the other vectors



a) with the same magnitude

$\vec{C}, \vec{D}$

b) with the same direction

$\vec{C}, \vec{E}$

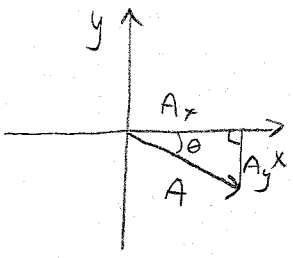
c) that are equal to **A**

$\vec{C}$

(3)

2. State the magnitude and direction of the following vector. Round your answers to two nearest decimal places. Show your work.

$A_x = 0.72$ m/s, $A_y = -0.16$ m/s



$A = \sqrt{A_x^2 + A_y^2}$

$= \sqrt{(0.72)^2 + (-0.16)^2}$

$= 0.737564$

$= 0.74$

$\tan\theta = \frac{A_y}{A_x}$

$= \frac{0.16}{0.72}$

$\theta = 12.5288°$

$= 12.53°$

but θ is in QIV, so angle in std position is either $-12.53°$ or $347.47°$ or coterminal

(3)

$A = 0.74$ m/s at $-12.53°$

3. Consider the vectors $\mathbf{A} = -3\mathbf{i} - \mathbf{j}$, $\mathbf{B} = -6\mathbf{i} + 2\mathbf{j}$

a) Find the dot product **A**·**B**, showing your work.

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$

$= (-3)(-6) + (-1)(2)$

$= 18 - 2 = 16$

(1)

b) Use your answer to part a) to calculate the angle between the two vectors.

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-6)^2 + (2)^2} = \sqrt{40}$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{16}{\sqrt{10}\sqrt{40}}$

$\theta = \cos^{-1}\left(\frac{16}{20}\right) = 36.8699 = 37°$

(3)

$\theta = 37°$

4. Find a unit vector **u** in the same direction as the given vector. Give exact answers and show your work.

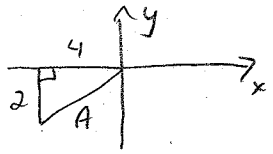
A = –4**i** – 2**j**

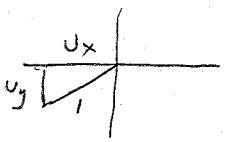
$A = \sqrt{A_x^2 + A_y^2}$

$= \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$





scaling down by $2\sqrt{5}$, $u_x = \frac{-4}{2\sqrt{5}} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$u_y = \frac{-2}{2\sqrt{5}} = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$$u = -\frac{2\sqrt{5}}{5}\hat{i} - \frac{\sqrt{5}}{5}\hat{j}$$

(3)

5. Calculate the magnitude of the vector **A** = 3**i** – 4**j** + 5**k** using the dot product. (Show your work!)

$A^2 = \vec{A} \cdot \vec{A} = A_x A_x + A_y A_y + A_z A_z$

$= 3^2 + (-4)^2 + 5^2$

$= 50$

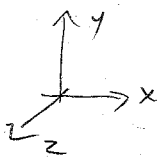
$A = \sqrt{50} = 5\sqrt{2}$

(3)

6. Vector **A** is in the negative *x*-direction, while vector **B** is in the *z*-direction.

a) What is the direction of **A**×**B**? + y

b) What is the direction of **B**×**A**? – y



(2)

7. Calculate the cross product **A**×**B** for the following vectors. Show your work.

A = – **i** + 6**j** – 2**k**, **B** = 4**j** + **k**

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 6 & -2 \\ 0 & 4 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -1 & 6 \\ 0 & 4 \end{matrix} = 6\hat{i} - 4\hat{k} - (-8\hat{i} - \hat{j})$$

$$= 6\hat{i} - 4\hat{k} + 8\hat{i} + \hat{j}$$

$$= 14\hat{i} + \hat{j} - 4\hat{k}$$

(3)

8. Solve the following systems of equations, using any **algebraic** method and showing your work

a) $\begin{cases} 12x-20y=-48 \\ -0.3x+0.5y=1.2 \end{cases}$ mult by 1/4, mult by 10

$3x - 5y = -12$
$-3x + 5y = 12$
$0 = 0$ ∴ same line

all points on the line
$3x-5y=-12$
(or equivalent)

(3)

b) $\begin{cases} 5x+2y=4 \\ 2x-3y=13 \end{cases}$ mult by 3, mult by 2

$15x + 6y = 12$
$4y - 6y = 26$
$19x = 38$
$x = 2$

sub into 1st
$5x + 2y = 4$
$10 + 2y = 4$
$2y = -6$
$y = -3$

(2, -3)

check: $2x - 3y = 13$
$4 - (-9) = 13$ ✓

(3)

c) $\begin{cases} x+y=4 \\ 2x-3z=14 \\ 2y+z=2 \end{cases}$

$x + y = 4$
$2x - 3z = 14$
$2y + z = 2$

mult by 2
mult by -1

$2x + 2y = 8$
$-2y - z = -2$
$2x - z = 6$

(1, 3, -4)

(5)

combine with 2nd eqn:

$2x - z = 6$ mult by -1
$2x - 3z = 14$

$-2x + z = -6$
$2x - 3z = 14$
$-2z = 8$
$z = -4$

sub back into
$2x - z = 6$
$2x + 4 = 6$
$2x = 2$
$x = 1$

sub into $x + y = 4$
$y = 3$

9. Marnie has 20 coins consisting of pennies, nickels, and dimes. The pennies and nickels together are worth fifty cents. The nickels and dimes together are worth $1.05. How many of each type of coin does Marnie have? You need to write out the system of equations, but may use a calculator to solve it if you wish. Write your answer in sentence form.

Let p = number of pennies
d = " " dimes
n = " " nickels

system:

$$p + d + n = 20$$
$$p + 5n = 50$$
$$10d + 5n = 105$$

augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 1 & 0 & 5 & 50 \\ 0 & 10 & 5 & 105 \end{array}\right]$$

↓ rref

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

(4)

Marnie has 5 pennies, 6 dimes, and 9 nickels.

10. Three alloys contain different percents of carbon, chromium, and iron. Alloy X is a type of wrought iron, alloy Y is a type of stainless steel, and alloy Z is a type of cast iron. How much of each of the three alloys can you make with 15 tons of carbon, 12.8 tons of chromium, and 392.2 tons of iron? You can solve this one using a calculator, but be sure to write out the augmented matrix and the resulting matrix in reduced row-echelon form. Give your answer in sentence form.

	X	*Y*	*Z*
Carbon	1%	1%	4%
Chromium	0%	14%	2%
Iron	99%	85%	94%

augmented matrix:

$$\left[\begin{array}{ccc|c} 0.01 & 0.01 & 0.04 & 15 \\ 0 & 0.14 & 0.02 & 12.8 \\ 0.99 & 0.85 & 0.94 & 392.2 \end{array}\right]$$

(4)

⇓ rref

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 360 \end{array}\right]$$

You can make 20, 40, and 360 tons of alloys X, Y, and Z, respectively.