Math 185 – Assignment #2 **Name:** Solution Set

Total: 40

1. State the values of x for which the functions below are continuous.

a) $f(x)=\sqrt{x+1}$ must be ≥ 0 so $x \geq -1$ — $x \geq -1$ or $[-1, \infty)$

b) $f(x)=x^3-2$ all values of x okay — $\mathbb{R}$ or $(-\infty, \infty)$ (3)

c) $f(x)=\begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x<0 \end{cases}$ this is just $f(x)=|x|$ — $\mathbb{R}$ or $(-\infty, \infty)$

note: when $x=0$, two lines intersect (no break)

2. Evaluate the following limits. Show your work.

a) $\lim\limits_{x \to 3} \sqrt{x^3-3x}$ — $3\sqrt{2}$ (4)

when $x=3$, $\sqrt{x^3-3x}$ exists

so $=\sqrt{27-9}$

$=\sqrt{18}$

$=3\sqrt{2}$

b) $\lim\limits_{x \to 3} \dfrac{x^2-3x}{3-x}$ — -3

$\dfrac{x^2-3x}{3x}$ doesn't exist at $x=3$

so $\lim\limits_{x \to 3} \dfrac{x^2-3x}{3-x} = \lim\limits_{x \to 3} \dfrac{x\cancel{(x-3)}}{-\cancel{(x-3)}} = \lim\limits_{x \to 3} -x$

$= -3$

3. Find the derivative of the function below using the definition.

$y=mx+b$

$\boxed{m}$ (3)

$\dfrac{dy}{dx} = \lim\limits_{h \to 0} \dfrac{y(x+h)-y(x)}{h}$

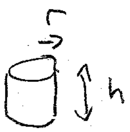
$= \lim\limits_{h \to 0} \dfrac{m(x+h)+b-(mx+b)}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{mx}+mh+\cancel{b}-\cancel{mx}-\cancel{b}}{h}$

$= \lim\limits_{h \to 0} \dfrac{m\cancel{h}}{\cancel{h}}$

$= \lim\limits_{h \to 0} m \quad \boxed{=m}$

4. An oil spill is increasing in size. For the purposes of this problem, treat the oil spill as if it were a cylinder of constant height (the thickness of the layer of oil) but with increasing radius. Find the instantaneous rate of change of the surface area A of the spill with respect to its radius r for $r = 1.5$ m.



$A = \underbrace{2(\pi r^2)}_{\text{circular parts}} + \underbrace{(2\pi r)h}_{\text{rectangular part}} = 2\pi r^2 + 2\pi r h$

$\frac{dA}{dr} = 2\pi(2r) + 2\pi h$

$= 4\pi r + 2\pi h$

at $r = 1.5$ m,

$\frac{dA}{dr} = 4\pi(1.5) + 2\pi h$

$= 6\pi + 2\pi h$

(and can substitute in for h if you wish)

③

5. Calculate the derivatives of the following functions.

a) $f(x) = 5x^7 - 4x^3 + 3$ $\qquad \frac{df}{dx} = 35x^6 - 12x^2$

b) $f(t) = -10t^{19} - 6t^8 + 35t$ $\qquad \frac{df}{dt} = -190t^{18} - 48t^7 + 35$

c) $f(r) = \frac{4\pi}{3}r^3 + \frac{1}{7}r^2 - 18$ $\qquad \frac{df}{dr} = 4\pi r^2 + \frac{2}{7}r$

③

6. Calculate the derivative of the following function at $x = -1$. Show your work.

$f(x) = 2x^4 - 5x^3 + 2x$ $\qquad f'(-1) = -21$

$f'(x) = 8x^3 - 15x^2 + 2$

$f'(-1) = 8(-1)^3 - 15(-1)^2 + 2$

$= -8 - 15 + 2$

$= -21$

②

7. Calculate the instantaneous velocity for an object moving with the following function for the displacement at the time $t = 2$ seconds.

$s = 80t - 16t^2$

②

$v = \frac{ds}{dt} = 80 - 32t$

at $t = 2$ seconds,

$v = 80 - 32(2) = 80 - 64 = 16$ (with unknown units, alas!)

8. Consider the two curves given by $y = 2x^2 - 5x$ and $y = 1 - 3x^2$. For what value or values of x do the tangent lines to these two curves have equal slopes?

line #1: line #2

$\frac{dy}{dx} = 4x - 5$ $\qquad$ $\frac{dy}{dx} = -6x$ $\quad$ 3

When the slopes are equal, the two derivatives are equal

$$4x - 5 = -6x$$
$$10x = 5$$
$$x = \tfrac{1}{2}$$

9. Calculate the following derivatives.

a) $f(x) = (3x^2 + 2)^6$

$$f'(x) = 6(3x^2+2)^5 \frac{d}{dx}(3x^2+2)$$ 2
$$= 6(3x^2+2)^5\, 6x$$
$$= 36x(3x^2+2)^5$$

b) $f(x) = \sqrt{3x - 10x^3}$ $= (3x - 10x^3)^{1/2}$

$$f'(x) = \tfrac{1}{2}(3x - 10x^3)^{-1/2} \frac{d}{dx}(3x - 10x^3)$$
$$= \tfrac{1}{2}(3x - 10x^3)^{-1/2}(3 - 30x^2)$$ 2
$$= \frac{(3 - 30x^2)}{2(3x - 10x^3)^{1/2}}$$

c) $f(T) = \frac{2T^2}{\sqrt[3]{1+4T}}$ $= \frac{2T^2}{(1+4T)^{1/3}}$

$$\frac{df}{dT} = \frac{(1+4T)^{1/3}4T - 2T^2\frac{1}{3}(1+4T)^{-2/3}\cdot 4}{(1+4T)^{2/3}}$$ 4

$$= \frac{1}{(1+4T)^{2/3}}\left[(1+4T)^{1/3}4T - \frac{8T^2}{3(1+4T)^{2/3}}\right]$$

$$= \frac{1}{(1+4T)^{2/3}}\left[\frac{(1+4T)4T\cdot 3}{3(1+4T)^{2/3}} - \frac{8T^2}{3(1+4T)^{2/3}}\right]$$

$$= \frac{1}{(1+4T)^{2/3}}\left[\frac{12T + 48T^2 - 8T^2}{3(1+4T)^{2/3}}\right] = \underbrace{\frac{12T + 40T^2}{3(1+4T)^{4/3}} = \frac{4T(3+10T)}{3(1+4T)^{4/3}}}_{\text{either}}$$

10. Find *dy/dx* by differentiating the following function implicitly. When applicable, express the result in terms of *x* and *y*.

$$\frac{d}{dx}\left(y^2x-\frac{5y}{x+1}+3x\right)=\frac{d}{dx}4$$

$$2yx\frac{dy}{dx}+y^2-\frac{(x+1)5\frac{dy}{dx}-5y}{(x+1)^2}+3=0$$

(4)

$$2yx\frac{dy}{dx}(x+1)^2+y^2(x+1)^2-(x+1)5\frac{dy}{dx}+5y+3(x+1)^2=0$$

$$2xy\frac{dy}{dx}(x+1)^2-5(x+1)\frac{dy}{dx}=-3(x+1)^2-y^2(x+1)^2-5y$$

$$\frac{dy}{dx}\left[2xy(x+1)^2-5(x+1)\right]=-(3+y^2)(x+1)^2-5y$$

$$\frac{dy}{dx}=\frac{-(3+y^2)(x+1)^2-5y}{(x+1)\left[2xy(x+1)-5\right]}=\frac{-(3+y^2)(x+1)^2-5y}{(x+1)(2x^2y+2xy-5)}$$

11. Find the derivative of the following function **using the definition**. Show your work.

$$f(x)=\sqrt{3x}$$

$$\frac{df(x)}{dx}=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}$$

(5)

$$=\lim_{h\to0}\frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}\left(\frac{\sqrt{3(x+h)}+\sqrt{3x}}{\sqrt{3(x+h)}+\sqrt{3x}}\right)$$

$$=\lim_{h\to0}\frac{3(x+h)-3x}{h\left(\sqrt{3(x+h)}+\sqrt{3x}\right)}$$

$$=\lim_{h\to0}\frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3(x+h)}+\sqrt{3x}\right)}$$

$$=\frac{3}{2\sqrt{3x}}$$

For what values of *x* is the function *f(x)* differentiable?

denom cannot equal zero
radical must have radicand ≥ 0

so $3x>0$
$x>0$