Math 185 – Assignment #3 **Name:** Solution Set

1. The cross section of a mirror is in the shape of a parabola, and the curve of the mirrored surface is given by $y^2 = -8x$. To figure out where light rays will be reflected when y is equal to 4, scientists need to study the line normal to the surface of the mirror at that point. Find the equation of this line.

5

$$y^2 = -8x$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(-8x)$$

$$2y\frac{dy}{dx} = -8$$

$$\frac{dy}{dx} = \frac{-8}{2y} \quad 1$$

$$\left.\frac{dy}{dx}\right|_{y=4} = \frac{-8}{2(4)} = -1 \quad 1$$

slope of tangent

$$m_{normal} = \frac{-1}{m_{tan}} = +1 \quad 1$$

at $y = 4$, $y^2 = -8x$
$16 = -8x$
$x = -2$ $(-2, 4)$ 1

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 1(x + 2)$$

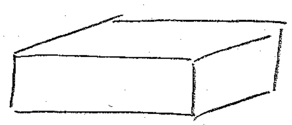
$$y = x + 2 + 4 \quad 1$$

$\boxed{y = x + 6}$ or $\boxed{x - y = -6}$

slope-intercept standard

2. The edges of a rectangular water tank are 3.00 m, 5.00 m, and 8.00 m. By Newton's method, determine by how much each edge should be increased equally to double the volume of the tank. (Note: you'll need a calculator for this problem!)

5



let x = amount added to each side

note: if you were more careful about choosing a starting point, could do fewer steps, but it still converges fast for this guess

$V_{old} = lwh = 3 \cdot 5 \cdot 8 = 120$

$V_{new} = (3+x)(5+x)(8+x)$ 1

$240 = (15 + 8x + x^2)(8+x)$

$240 = 120 + 15x + 64x + 8x^2 + 8x^2 + x^3$ 1

$f(x) = 0 = x^3 + 16x^2 + 79x - 120$

$f'(x) = 3x^2 + 32x + 79$ 1

x	$f(x)$	$f'(x)$	$x - \frac{f(x)}{f'(x)}$
3	288	202	1.57426
1.57426	47.9204	136.811	1.22399
1.22399	2.49944	122.662	1.20361
1.20361	0.008159	121.862	1.20355
1.20355			

agree to 3rd decimal which is more accurate than original measurements (2 decimal places)

note: if you instead multiplied each side by the same constant, should find $x = 1.25992 \left(= \sqrt[3]{2}\right)$ instead

You must add 1.20 m to each side to double the volume.

3. A roller mechanism follows a path described by $y = \sqrt{4x+1}$, where displacements are in m and velocities in m/s. If $v_x = 3x$, find the **magnitude** of the resultant velocity for $x = 2$ m. You may leave your answer in radical form or give a decimal approximation, your choice.

$$v_y = \frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} \quad (1)$$

$$y = \sqrt{4x+1}$$

$$\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4) = \frac{2}{\sqrt{4x+1}} \quad (1)$$

$$= \frac{2}{\sqrt{4x+1}}\,3x$$

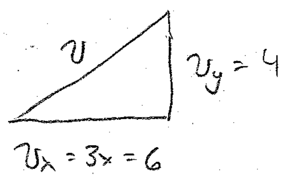
$$\frac{dx}{dt} = v_x = 3x \quad (1)$$

(5)

$$= \frac{6x}{\sqrt{4x+1}}$$

note
$(-\frac{1}{2})$ if $\sqrt{52}$ and not simplified
$(-\frac{1}{2})$ no units

$$= \frac{12}{\sqrt{4\cdot 2+1}} = 4 \quad \text{at } x=2 \quad (1)$$



$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52} = 2\sqrt{13} \quad (1)$$

$\boxed{\text{The velocity has a magnitude of } 2\sqrt{13} \text{ m/s at } x = 2\text{m.}}$

4. A metal sphere is placed in seawater to study the corrosive effect of seawater. If the surface area **decreases** at 240 cm^2/year due to corrosion, how fast is the radius changing when it is 30 cm? (You may leave your answer in terms of π if you wish.)

note: (-1) if no units

$$A_{sphere} = 4\pi r^2$$

$$\frac{dA}{dr} = 8\pi r \quad (1)$$

(4)

$$\frac{dA}{dt} = \frac{dA}{dr}\,\frac{dr}{dt} \quad (1)$$

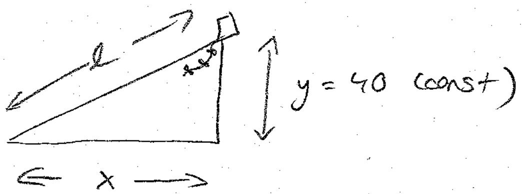
$$\text{so} \quad \frac{dr}{dt} = \frac{dA}{dt}\Big/\frac{dA}{dr} = \frac{-240}{8\pi r} = \frac{-240}{8\pi\, 30} = -\frac{1}{\pi} \approx -0.318 \text{ cm/year} \quad (1) \quad (1)$$

The radius is changing at $\left(-\frac{1}{\pi}\right)$ cm/year.

[or decreasing at $\frac{1}{\pi}$ cm/year also okay]

5. A child is flying a kite on a windy day. Assume that the kite maintains a constant height of 40 meters above the ground while moving away from the child at a rate of 1 m/s relative to the ground. At what rate is the string being let out when 50 meters of string are already out?

$\boxed{(-1) \text{ for no units}}$



by Pythagorus,

$x^2 + y^2 = l^2$ (1)

$x^2 + 40^2 = l^2$

$2x \frac{dx}{dt} = 2l \frac{dl}{dt}$ (1)

$\frac{dl}{dt} = \frac{x}{l} \frac{dx}{dt}$ (1)

when $l = 50$, $x = 30$,

$\frac{dx}{dt} = 1$ m/s (1)

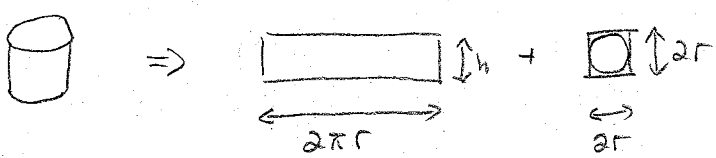
$\frac{dl}{dt} = \frac{30}{50}(1 \text{ m/s})$ (1)

$= \frac{3}{5}$ m/s (or 0.6 m/s)

The string is being let out at 0.6 m/s.

(5)

6. A cylindrical cup (no top) is designed to hold 500 cm^3 (500 mL) of liquid. There is no waste in the material used for the sides. However, there is waste in that the bottom of the cup is made from a square $2r$ on a side. What are the most economical dimensions for a cup made under these conditions?



$V = \pi r^2 h$

$500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$

$A = 2\pi r h + (2r)^2$ (1)

$A = 2\pi r \left(\frac{500}{\pi r^2}\right) + 4r^2$

$A = \frac{1000}{r} + 4r^2$ (1)

$0 = \frac{dA}{dr} = -\frac{1000}{r^2} + 8r$ (1)

$\frac{1000}{r^2} = 8r$

$r^3 = \frac{1000}{8}$

$r = \sqrt[3]{\frac{1000}{8}} = \frac{10}{2} = 5$ (1)

check for max/min

$\frac{d^2A}{dr^2} = \frac{+2000}{r^3} + 8$ (1)

When r is +, this is +

concave up

∴ min point ✓

$h = \frac{500}{\pi r^2} = \frac{500}{\pi(25)} = \frac{20}{\pi}$ (1)

The most economical dimensions are 5cm for the radius and $20/\pi$ cm ($\approx$ 6.4 cm) for the height.

(6)

7. Consider the function $y = \frac{x^2 - 1}{x^2 - 4}$.

a) Find the x- and y- intercepts, if any.

1

y-int: set $x=0$ $y = \frac{-1}{-4} = \frac{1}{4}$ $(0, \frac{1}{4})$

2

x-int set num $= 0$ $x^2 - 1 = 0$ $(1, 0)$ 1 and $(-1, 0)$

$x^2 = 1$

$x = \pm 1$

b) Find the equations of the asymptotes, if any.

as $x \to \pm\infty$ $y \to \left(\frac{x^2}{x^2} = 1\right)$ so horizontal asymptote at $\boxed{y=1}$

vertical asymptotes at places where denom $= 0$ 2

$x^2 = 4$

$\boxed{x = \pm 2}$ vertical

c) Find any critical values, if any. If there are critical points, use one of the derivative tests to verify whether these points are relative maxima or minima and show your work.

$$\frac{dy}{dx} = \frac{(x^2-4)2x - (x^2-1)2x}{(x^2-4)^2}$$

$$= \frac{2x^3 - 8x - 2x^3 + 2x}{(x^2-4)^2}$$

5

$$= \frac{-6x}{(x^2-4)^2}$$

$$0 = \frac{-6x}{(x^2-4)^2}$$ 2

$x = 0$ 1 there is a critical point at $x=0$

$$\frac{d^2y}{dx^2} = \frac{(x^2-4)^{\cancel{2}}(-6) - (-6x)\cancel{(x^2-4)}2(2x)}{(x^2-4)^{\cancel{4}}}$$

1

$$= \frac{-6(x^2-4) + 24x^2}{(x^2-4)^3} = \frac{18x^2+24}{(x^2-4)^3}$$ 1 $= -$ when $x = 0$

concave down

$\therefore$ max

d) Find the inflection points of this function, if any.

$$0 = \frac{d^2y}{dx^2} = \frac{18x^2+24}{(x^2-4)^3} \quad (1)$$

$$0 = 18x^2+24$$

$$x^2 = -\frac{24}{18} \leftarrow \text{no real sol'ns} \quad (1)$$

$\therefore$ no inflection points (1)

(3)

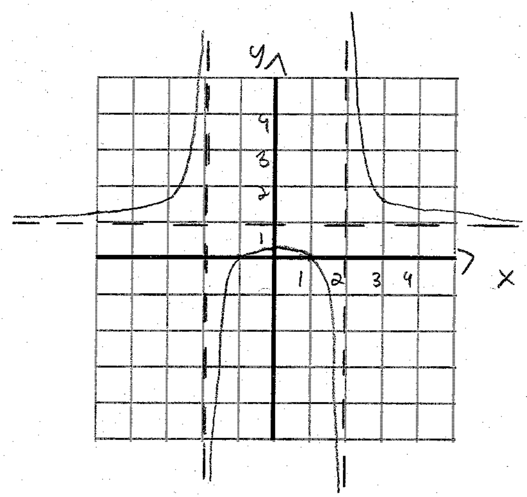
e) Sketch the resulting graph.

(1) asymptotes

(2) correct curves

(−½) for no x, y labels

(−½) for no scales



(3)

8. Find the differential dy for $y = \sqrt{\dfrac{x}{1+2x}}$.

$$y = \left(\frac{x}{1+2x}\right)^{1/2} \quad (1)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{1+2x}\right)^{-1/2} \frac{d}{dx}\left(\frac{x}{1+2x}\right) \quad (2)$$

$$= \frac{1}{2}\left(\frac{1+2x}{x}\right)^{1/2} \frac{(1+2x)(1) - x(2)}{(1+2x)^2}$$

$$= \frac{1}{2}\sqrt{\frac{1+2x}{x}} \; \frac{1+2x-2x}{(1+2x)^2} \quad (1)$$

$$= \frac{1}{2}\sqrt{\frac{1+2x}{x}} \; \frac{1}{(1+2x)^2} \quad \text{or} \quad \frac{1}{2x^{1/2}(1+2x)^{3/2}}$$

$$dy = \frac{1}{2}\sqrt{\frac{1+2x}{x}} \; \frac{1}{(1+2x)^2}\,dx \quad (1)$$

(5)