Math 185 – Assignment #4 **Name:** Solution Set

Total: 40

1. Find the derivative $\frac{dy}{dx}$ for the following functions.

a) $y = \left(1 - \sin^2 x\right)^4$

$$\frac{dy}{dx} = 4(1-\sin^2 x)^3 \frac{d}{dx}(1-\sin^2 x)$$

$$= 4(1-\sin^2 x)^3 (-2\sin x)(\cos x)$$

$$= -8 \sin x \cos x (1-\sin^2 x)^3$$

(3)

(note: can simplify further if you notice that $1-\sin^2 x = \cos^2 x$, so $\frac{dy}{dx} = -8 \sin x \cos^7 x$)

b) $y = \cos^{-1}\left(5x^3\right)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(5x^3)^2}} 15x^2$$

$$= \frac{-15x^2}{\sqrt{1-25x^6}}$$

(3)

c) $y = \left(e^{3/x} \tan x\right)^2$

$$\frac{dy}{dx} = 2(e^{3/x} \tan x)\left(e^{3/x} \sec^2 x + \tan x \, e^{3/x}\left(-\frac{3}{x^2}\right)\right)$$

(4)

$$= 2 e^{3/x} (\tan x) e^{3/x} \left(\sec^2 x - \frac{3\tan x}{x^2}\right)$$

$$= 2 e^{6/x} \tan x \left(\sec^2 x - \frac{3 \tan x}{x^2}\right)$$

d) $x\cos 2y + \sin x \cos y = 1$

$$x(-\sin 2y)\,2\frac{dy}{dx} + \cos 2y + \sin x(-\sin y)\frac{dy}{dx} + \cos x \cos y = 0$$

$$-2x \sin 2y \frac{dy}{dx} - \sin x \sin y \frac{dy}{dx} = -\cos x \cos y - \cos 2y$$

$$\frac{dy}{dx} = \frac{\cos x \cos y + \cos 2y}{2x \sin 2y + \sin x \sin y}$$ 5

2. Find the derivative $f'(x)$ for the following functions. These ones should simplify nicely.

a) $f(x) = x\cos^{-1} x - \sqrt{1-x^2}$

$$f'(x) = x \cdot \frac{(-1)}{\sqrt{1-x^2}} + \cos^{-1} x \cdot 1 - \left(+\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$$ 3

$$= \frac{-x}{\sqrt{1-x^2}} + \cos^{-1} x + \frac{x}{\sqrt{1-x^2}}$$

$$= \cos^{-1} x$$

b) $f(x) = \ln(\cos x)$

$$f'(x) = \frac{1}{\cos x}(-\sin x)$$ 2

$$f'(x) = -\tan x$$

3. Rewrite $y = \sec x$ in terms of the basic trig functions $\sin x$ and/or $\cos x$ and then differentiate in order to **derive** the rule for differentiating the secant function. (At least here, you know what the answer will be!)

$$y = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$$ 3

$$= \frac{\sin x}{\cos^2 x} \quad \leftarrow \text{perfectly acceptable answer}$$

$$= \tan x \sec x \quad \leftarrow \text{also perfectly acceptable}$$

$$\therefore \quad \frac{d}{dx}(\sec \square) = \tan \square \sec \square \frac{d\square}{dx}$$

4. A block sitting on a frictionless tabletop is attached to a spring and given a push. The block then oscillates back and forth, with the horizontal displacement given by $x = 5\sin(2\pi t)$, where x is in centimetres, t is in seconds, and the product $2\pi t$ has units of radians. Find the velocity and acceleration of the block at $t = 1$ second. 4

$$x = 5\sin(2\pi t)$$

at $t = 1$ s

$$x = 5\sin 2\pi = 0$$

$$v_x = \frac{dx}{dt} = 5\cos(2\pi t)\cdot 2\pi$$

$$v_x = 10\pi \cos 2\pi t$$

$$\boxed{\begin{array}{l}\text{at } t = 1\text{s} \\ v = 10\pi \cos 2\pi = 10\pi \text{ cm/s}\end{array}}$$

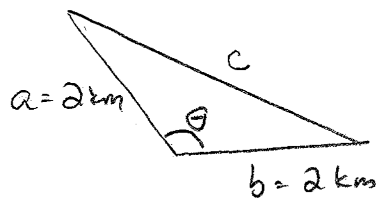
$$a_x = \frac{d^2x}{dt^2} = 10\pi(-\sin 2\pi t)\cdot 2\pi$$

$$a_x = -20\pi^2 \sin 2\pi t$$

at $t = 1$s

$$\boxed{\begin{array}{l}a_x = -20\pi^2 \sin 2\pi \\ = 0\end{array}}$$

5. A surveyor measures two sides and the included angle of a triangular parcel of land to be 2 km, 2 km, and 120°. Use a differential to estimate the error in determining the length of the third side if the angle has an error of 1°.



$\theta = 120° \pm 1°$

cosine law

7

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 4 + 4 - 8\cos\theta$

$c^2 = 8 - 8\cos\theta$ 1 $\Rightarrow$ implicit diff: 1

$\frac{d}{d\theta}(c^2) = \frac{d}{d\theta}(8 - 8\cos\theta)$

$2c\,\frac{dc}{d\theta} = -8(-\sin\theta)$

$\frac{dc}{d\theta} = \frac{8\sin\theta}{2c}$

$= \frac{4\sin\theta}{c}$ 1

when $\theta = 120°$, $\cos\theta = -\frac{1}{2}$

$c^2 = 8 - 8(-\frac{1}{2})$

$= 8 + 4 = 12$

$c = \sqrt{12} = 2\sqrt{3}$ 1

at $\theta = 120°$, $\sin\theta = \sqrt{3}/2$ 1

$\frac{dc}{d\theta} = \frac{4\sqrt{3}}{2 \cdot 2\sqrt{3}} = 1$

1

$dc = \frac{dc}{d\theta} \cdot d\theta$

$= 1 \cdot 1° \left(\frac{\pi}{180°}\right)$

$= \frac{\pi}{180}$ km

$\boxed{dc \approx 0.0174 \text{ km}}$

1

if you like, you can check:

$C_{max} = \sqrt{8 - 8\cos(121°)} \approx 3.48142$

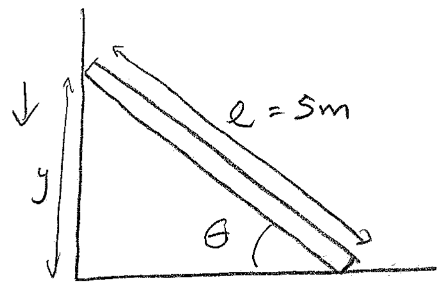
$C_{min} = \sqrt{8 - 8\cos(119°)} \approx 3.44652$

$\Delta C = \frac{1}{2}(C_{max} - C_{min}) = 0.017453$ km

same result!

6. A ladder is slipping down a vertical wall. The ladder is 5 m long and the top of it is slipping down the wall at a constant rate of 0.3 m/s. Let θ be the angle that the ladder makes with the ground. When θ equals 53.1°, at what rate is θ changing? You may leave your answer in rads/second if you wish.

(For ease of calculation, use sin 53.1° ≈ 4/5 and cos 53.1° ≈ 3/5.)



$\sin\theta = \frac{y}{l}$

$y = l \sin\theta$

$y = 5\sin\theta$

differentiate wrt t:

$\frac{dy}{dt} = 5\cos\theta \frac{d\theta}{dt}$

$\frac{dy}{dt} = -0.3$ m/s

$\frac{d\theta}{dt} = \frac{1}{5\cos\theta}\frac{dy}{dt}$

$= \frac{1}{5(3/5)}(-0.3)$

$= -0.1$ rads/s

(6)

θ is <u>decreasing</u> at 0.1 rads/s