Math 185 – Assignment #5 **Name:** Solution Set

Total : 30

1. Find, if possible, A+B, AB, and BA. If the result is undefined, say so.

a) $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}-3 & -2\\ 4 & 2\end{bmatrix}$

$A+B=\begin{bmatrix}-2 & 0\\ 6 & 3\end{bmatrix}$ (1)

$AB=\begin{bmatrix}5 & 2\\ -2 & -2\end{bmatrix}$ (1)

$BA=\begin{bmatrix}-7 & -8\\ 8 & 10\end{bmatrix}$ (1)

b) $A=\begin{bmatrix}3\\ 2\\ -1\end{bmatrix}, B=\begin{bmatrix}-4 & 6 & 2\end{bmatrix}$

$A+B=$ undefined (1)

$AB=\begin{bmatrix}-12 & 18 & 6\\ -8 & 12 & 4\\ 4 & -6 & -2\end{bmatrix}$ (1)

$BA=[-2]$ (1)

c) $A=\begin{bmatrix}1 & 2 & 0\\ -1 & 0 & 3\end{bmatrix}, B=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$A+B=$ undefined (1)

$AB=$ undefined (can't multiply 2×3 and 2×2) (1)

$BA=\begin{bmatrix}2 & 6 & 3\\ -3 & -2 & 6\end{bmatrix}$ (1)

2. Find the inverse of the following matrices (if the inverse exists).

a) $\begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$

$\text{inverse} = \frac{1}{13}\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$ (1)

b) $\begin{bmatrix} 0.1 & 0.6 \\ -2 & -12 \end{bmatrix}$

inverse does not exist (determinant = 0) (1)

c) $\begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix}$

$\text{inverse} = \begin{bmatrix} \frac{1}{2} & -2 & -\frac{11}{2} \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix}$ (1)

d) $\begin{bmatrix} 3 & 5 & 8 \\ 1 & 0 & 1 \end{bmatrix}$

inverse does not exist (not square) (1)

3. Use an inverse matrix to solve, if possible, the following systems of linear equations. Show your work!

a) $-x + y = 4$
$2x - 4y = -34$

$\begin{bmatrix} -1 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -34 \end{bmatrix}$

$AX = B$

$X = A^{-1}B$

$= \begin{bmatrix} 9 \\ 13 \end{bmatrix}$

$(x, y) = (9, 13)$ (3)

inverse of $A = A^{-1} = \begin{bmatrix} -2 & -\frac{1}{2} \\ -1 & -\frac{1}{2} \end{bmatrix}$

b) $x - 3y = 5$
$-2x + 6y = -10$

$\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$

$AX = B$

$X = A^{-1}B$

all points on line $x - 3y = 5$ (3)

inverse of $A = A^{-1}$ = does not exist

check: the second line is the same as the first

∴ all points on line

$$2x+4y+5z=3$$
$$y+4z=-1$$

c) $-z=4$ (hint: use your answer for 2c) $(-37/2, 15, -4)$

(3)

$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} 1/2 & -2 & -11/2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} -37/2 \\ 15 \\ -4 \end{bmatrix}$$

$$x-3y+2z=-11$$
$$2x-4y+3z=-15$$

d) $3x-5y-4z=5$ $(1, 2, -3)$

(3)

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & -4 & 3 \\ 3 & -5 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11 \\ -15 \\ 5 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} -31/16 & 11/8 & 1/16 \\ -17/16 & 5/8 & -1/16 \\ -1/8 & 1/4 & -1/8 \end{bmatrix}\begin{bmatrix} -11 \\ -15 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

4. Find the determinant of the following matrices.

a) $\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 8 \end{bmatrix}$

8 (1)

b) $\begin{bmatrix} -2 & 1 & 3 \\ -1 & 4 & 2 \\ 2 & 1 & 2 \end{bmatrix}$

-33 (1)

5. Solve the following system using Cramer's Rule and show your work!

$0.6x + 0.5y = 19$
$0.5x - 0.25y = 7$

(20.625, 13.25) (3)

$D = \begin{vmatrix} 0.6 & 0.5 \\ 0.5 & -0.25 \end{vmatrix} = -0.4$ (1)

$D_x = \begin{vmatrix} 19 & 0.5 \\ 7 & -0.25 \end{vmatrix} = -8.25$

$D_y = \begin{vmatrix} 0.6 & 19 \\ 0.5 & 7 \end{vmatrix} = -5.3$ (1)

$x = \frac{D_x}{D} = \frac{-8.25}{-0.4} = 20.625$

$y = \frac{-5.3}{-0.4} = 13.25$ (1)