

# Math 185 – Quiz #1

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Name: Solution Set

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Total: 40 points

1. Which of the following are vectors or scalars? Write “V” for vector, “S” for scalar, and “neither” if the quantity isn’t a vector or a scalar. (4 points)

- a) 15 m/s north
- b) 15° above the horizontal
- c) 8
- d) 14.53 seconds

✓
neither
S
S

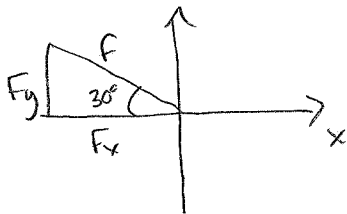
2. Vector **A** is in the +z-direction, while vector **B** is in the +y-direction. (1 point)

What is the direction of  $\vec{A} \times \vec{B}$ ?

negative x-direction

3. Calculate the component form of the following vector. Give exact answers in **ijk** notation and show your work. (4 points)

$F = 12$  at  $150^\circ$



$$\sin 30^\circ = \frac{F_y}{F}$$

$$\begin{aligned} F_y &= F \sin 30^\circ \\ &= 12 \left(\frac{1}{2}\right) \\ &= 6 \end{aligned}$$

$$\begin{aligned} F_x &= F \cos 30^\circ \\ &= 12 \left(\frac{\sqrt{3}}{2}\right) \\ &= 6\sqrt{3} \end{aligned}$$

$$F_x = -6\sqrt{3}$$

← but component should be - from diagram so

$$\vec{F} = -6\sqrt{3} \hat{i} + 6 \hat{j}$$

4. Calculate the magnitude of the vector  $\mathbf{R} = -5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$  using the dot product. Show your work and give an exact answer. (3 points)

$$\begin{aligned} R^2 &= \vec{R} \cdot \vec{R} = R_x R_x + R_y R_y + R_z R_z \\ &= (-5)^2 + (-2)^2 + 4^2 \\ &= 45 \end{aligned}$$

$$R = \sqrt{45} = 3\sqrt{5}$$

5. Consider the vectors  $\mathbf{A} = \mathbf{i} - \mathbf{k}$ ,  $\mathbf{B} = \mathbf{j} + \mathbf{k}$

(4 points)

a) Find the dot product  $\mathbf{A} \cdot \mathbf{B}$ , showing your work.

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z \\ &= 1 \cdot 0 + 0 \cdot 1 + (-1)(1) \\ &= -1\end{aligned}$$

vector arrows missing

 $-\frac{1}{2}$ 

resultant as vector

 $-1$ 

missed that it's 30

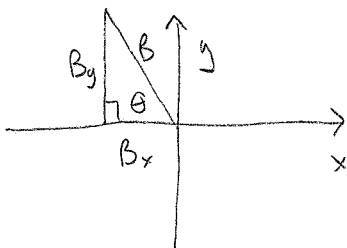
 $-2$ 

b) Are these two vectors perpendicular? Explain.

No, because for  $\vec{A}$  and  $\vec{B}$  to be  $\perp$ ,  $\vec{A} \cdot \vec{B}$  would have to equal zero.

6. Calculate the magnitude and direction of the following vector. Round your answers to the nearest decimal and show your work. (3 points)

$$B_x = -10.2, B_y = 14.3$$



$$\begin{aligned}B &= \sqrt{B_x^2 + B_y^2} \\ &= \sqrt{(-10.2)^2 + (14.3)^2} \\ &= 17.565 = 17.6\end{aligned}$$

① mag

② direction

① if wrong  
Qued

$$\tan \theta = \frac{B_y}{B_x} = \frac{14.3}{10.2}$$

$$\theta = 54.5^\circ$$

but we want a QIII angle (standard position), so

$$\vec{B} = 17.6 \text{ at } +125.5^\circ$$

7. Calculate the cross product  $\mathbf{A} \times \mathbf{B}$  for the following vectors. Show your work. (3 points)

$$\mathbf{A} = -5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}, \mathbf{B} = \mathbf{j}$$

$$\begin{aligned}\vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -2 & 3 \\ 0 & 1 & 0 \end{vmatrix} = 0\hat{i} + 0\hat{j} - 5\hat{k} - (0\hat{k} + 3\hat{i} + 0\hat{j}) \\ &= -5\hat{k} - 3\hat{i} \\ &= -3\hat{i} - 5\hat{k}\end{aligned}$$

8. Solve the following systems of equations, using any **algebraic** method and showing your work (6 points)

a) 
$$\begin{cases} -2x + 3y = 6 & \text{mult by 3} \\ 3x - 5y = -11 & \text{mult by 2} \end{cases}$$

(3, 4)

$$\begin{array}{r} -6x + 9y = 18 \\ 6x - 10y = -22 \\ \hline -y = -4 \\ y = 4 \end{array}$$

check:

$$\begin{array}{l} 3x - 5y = -11 \\ 9 - 20 = -11 \checkmark \end{array}$$

$$\begin{array}{l} -2x + 3y = 6 \\ -2x + 12 = 6 \\ -2x = -6 \\ x = 3 \end{array}$$

b) 
$$\begin{cases} y = 5 + \frac{1}{2}x \\ 2x - 4y = 7 \end{cases}$$

$\emptyset$

$$2x - 4\left(5 + \frac{1}{2}x\right) = 7$$

$$2x - 20 - 2x = 7$$

$$-20 = 7$$

$\therefore$  no solutions

⊖ if soln  
"all points on line"

⊖ if say "parallel"  
but no info  
on solns

9. Write the augmented matrix associated with the following system of equations. Then write the reduced-row echelon form of the matrix, and finally write the solution to the system as an ordered triple. (3 points)

$$\begin{cases} 3x - 4y + z = 19 \\ 2x + 4y + z = 0 \\ x + 5z = 13 \end{cases}$$

augmented:

$$\left[ \begin{array}{ccc|c} 3 & -4 & 1 & 19 \\ 2 & 4 & 1 & 0 \\ 1 & 0 & 5 & 13 \end{array} \right]$$

(3, -2, 2)

ref:

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$\uparrow$   
 $\leftarrow$  ① each

10. For the following two word problems, write out the system of equations, solve using any method (if using a calculator, write out the associated matrices), and state your answer in sentence form.

- a) At the neighbourhood café, a coffee and a doughnut together cost \$2.75. If the price of the doughnut is twenty-five cents less than three times the price of the coffee, how much does the café charge for each item? (4 points)

let  $c$  = cost of coffee (in cents)

$d$  = cost of doughnut (" )

$$c + d = 275$$

$$d = 3c - 25$$

substitute:

$$c + (3c - 25) = 275$$

$$4c = 300$$

$$c = 75$$

$$\begin{aligned} d &= 3c - 25 \\ &= 225 - 25 \\ &= 200 \end{aligned}$$

A coffee is \$0.75  
and a doughnut  
costs \$2.00 at  
the café.

- b) Every year, the Times Colonist holds a big book sale for charity. All hardcover books sell for the same price, all paperbacks sell for another price, and all magazines have a third price. Edgar buys three hardcovers, one paperback, and four magazines for \$4.50. Cynthia buys four paperbacks and two magazines for \$2.50. Herbert buys one hardcover, two paperbacks, and two magazines for \$2.50. How much must I pay in total if I want to buy one hardcover, one paperback, and one magazine? (5 points)

let  $h$  = cost of hardcover

$p$  = " " paperback

$m$  = cost of magazine

$$3h + p + 4m = 4.50$$

$$4p + 2m = 2.50$$

$$h + 2p + 2m = 2.50$$

augmented

$$\left[ \begin{array}{ccc|c} 3 & 1 & 4 & 4.50 \\ 0 & 4 & 2 & 2.50 \\ 1 & 2 & 2 & 2.50 \end{array} \right]$$

ref:

$$\left[ \begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0.25 \end{array} \right]$$

$$\text{so } h = \$1.00$$

$$p = \$0.50$$

$$m = \$0.25$$

$$\text{total} = \$1.75$$

I must pay \$1.75  
for one of each item.