Math 185 – Quiz #2

November 2, 2007 **Name:** Solution Set
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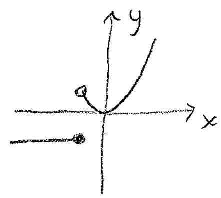
Total: 40 points

1. State the values of x for which the functions below are continuous. (4 points)

a) $f(x) = \frac{1}{x}$

$\{x \mid x \neq 0\}$
or $(-\infty, 0) \cup (0, \infty)$
or $x > 0$ or $x < 0$

b) $f(x) = \begin{cases} x^2 & \text{for } x > -1 \\ -1 & \text{for } x \le -1 \end{cases}$



$\{x \mid x \neq -1\}$
or $(-\infty, -1) \cup (-1, \infty)$

$(-\frac{1}{2})$ for $x \le -1$, $x \ge -1$

(-1) for $\frac{1}{2}$ of graph only

(-1) if correct graph given

2. Evaluate the following limit. Show your work. (3 points)

$$\lim_{x \to -2} \frac{2+x}{4-x^2} = \lim_{x \to -2} \frac{\cancel{2+x}}{(2-x)\cancel{(2+x)}}$$

$$= \lim_{x \to -2} \frac{1}{(2-x)}$$

$$= \frac{1}{4}$$

$\frac{1}{4}$

3. Calculate the slope of the line tangent to the following curve at $x = 10$. Show your work. (3 points)

$f(x) = x^4 - 5x^2 + 25$

$f'(10) = 3900$

$f'(x) = 4x^3 - 10x$

$f'(10) = 4 \cdot 10^3 - 10^2$

$= 4000 - 100$

$= 3900$

(could also use $\frac{df(x)}{dx}$ and $\left.\frac{df(x)}{dx}\right|_{x=10}$ notation)

$(-\frac{1}{2})$ missing $f'(x)$ and $\frac{dy}{dx}$

$(-\frac{1}{2})$ answer as (x, y)

4. Using the **definition of derivative**, find the derivatives of the functions below with respect to x. Show your work. (8 points)

a) $y = 5 - 2x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h}$$

$$= \lim_{h \to 0} \frac{5 - 2(x+h) - (5 - 2x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5} - \cancel{2x} - 2h - \cancel{5} + \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}}$$

$$= -2$$

missing "$\lim_{h \to 0}$" (-1) for both

(3) { (1) definition
(1) subs for $y(x+h)$, $y(x)$
(1) simplification
give ($\frac{1}{2}$) if only answer

b) $f(x) = (x+1)^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h+1)^2 - (x+1)^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + xh + x + xh + h^2 + h + x + h + 1 - (x^2 + 2x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + \cancel{2x} + 2h + h^2 + \cancel{1} - \cancel{x^2} - \cancel{2x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + 2 + h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + 2 + h$$

$$= 2x + 2$$

(4)

(2) subst for $f(x+h)$, $f(x)$
(1) expansion of $(x+h+1)^2$
(1) simplification
give ($\frac{1}{2}$) for answer only

c) For what values of x is the function $f(x)$ in part (b) differentiable?

all real

or $\mathbb{R}$

or $(-\infty, \infty)$

(-1)

5. Consider the curve given by the function $f(x)=\frac{-x}{x^2+9}$. For what values of x (if any) does the curve have a horizontal tangent line? (4 points)

$$f'(x) = \frac{(x^2+9)(-1)+(-x)2x}{(x^2+9)^2}$$

$$= \frac{x^2+9-2x^2}{(x^2+9)^2}$$

$$= \frac{-x^2+9}{(x^2+9)^2} \quad 2$$

tangent is horizontal when $f'(x)=0$

$$0 = \frac{-x^2+9}{(x^2+9)^2} \quad 1$$

$$0 = -x^2+9$$

$$x^2 = 9$$

$$\boxed{x = \pm 3} \quad 1$$

missing $\pm$ (-1)

6. Calculate the instantaneous velocity at $t = 2$ seconds for an object moving with the following function for the displacement (s is in metres). (4 points)

$$s=\sqrt{(t+1)^2+7}$$

(-½) missing or incorrect units

$$v = \frac{ds}{dt} = \frac{1}{\cancel{2}}\frac{\cancel{2}(t+1)}{\sqrt{(t+1)^2+7}} = \frac{t+1}{\sqrt{(t+1)^2+7}} \quad 2$$

at $t=2$

$$v = \frac{3}{\sqrt{9+7}}$$

$$= \frac{3}{\sqrt{16}} = 3/4 \quad 2$$

$v = 3/4$ m/s at $t=2$

(or 0.75 m/s)

7. Find the slope of the line tangent to the curve of $2y^3+xy+1=0$ at the point (–3,1). (4 points)

implicit differentiation:

$$\frac{d}{dx}(2y^3+xy+1) = \frac{d}{dx}(0)$$

$$6y^2\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$(6y^2+x)\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{6y^2+x}$$

$$= \frac{-1}{6-3}$$

$$\boxed{\frac{dy}{dx} = -1/3}$$

1 implicit diff
1 correct product rule
1 simplification
1 evaluation

8. Calculate the following derivative with respect to x. To receive full marks, write as a single fraction and simplify fully. (4 points)

$y=(2x+1)\sqrt[3]{x+3} = (2x+1)(x+3)^{1/3}$

$\frac{dy}{dx} = (2x+1)\frac{1}{3}(x+3)^{-2/3} + (x+3)^{1/3}(2)$

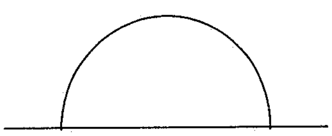
$= \frac{2x+1}{3(x+3)^{2/3}} + 2(x+3)^{1/3}\frac{3(x+3)^{2/3}}{3(x+3)^{2/3}}$ (Lowest common denominator)

$= \frac{2x+1 + 6(x+3)}{3(x+3)^{2/3}}$

$= \frac{2x+1+6x+18}{3(x+3)^{2/3}} = \frac{8x+19}{3(x+3)^{2/3}}$

① product rule
① differentiate radical
① chain rule → 2
① simplify

9. A drop of water in the shape of a hemisphere (half of a sphere) is sitting on a flat counter as in the diagram below. As the water in the drop evaporates, the radius r of the drop changes. Find the instantaneous rate of change $\frac{dV}{dr}$ for the volume V of the drop and also the rate of change $\frac{dA}{dr}$ for the **total** surface area A. Then evaluate these rates of change for $r = 1$ mm. You may leave your answer in terms of π. (6 points)



$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{hemisphere} = \frac{2}{3}\pi r^3$

$\frac{dV}{dr} = \frac{2}{3}\cdot 3\pi r^2$

$= 2\pi r^2$

$\left.\frac{dV}{dr}\right|_{r=1mm} = 2\pi \ mm^2$

↑ could also have $\frac{mm^3}{mm}$ as well

$A_{sphere} = 4\pi r^2$

$A_{hemisphere} = \underbrace{\tfrac{1}{2}(4\pi r^2)}_{\text{round}} + \underbrace{\pi r^2}_{\text{flat}}$

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$\frac{dA}{dr} = 3\pi(2r)$

$= 6\pi r$

$\left.\frac{dA}{dr}\right|_{r=1mm} = 6\pi \ mm$